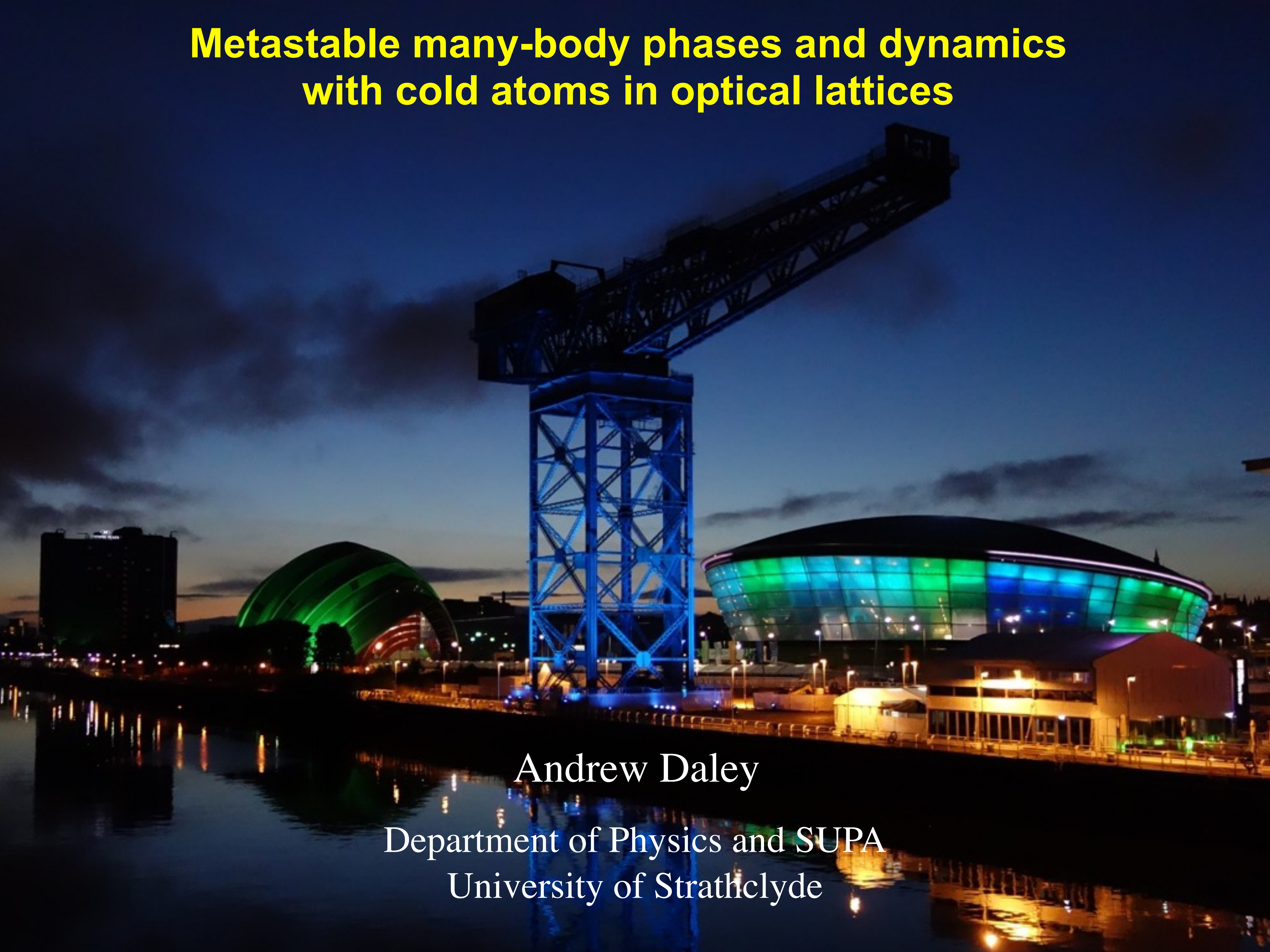
Metastable many-body phases and dynamics with cold atoms in optical lattices



Andrew Daley

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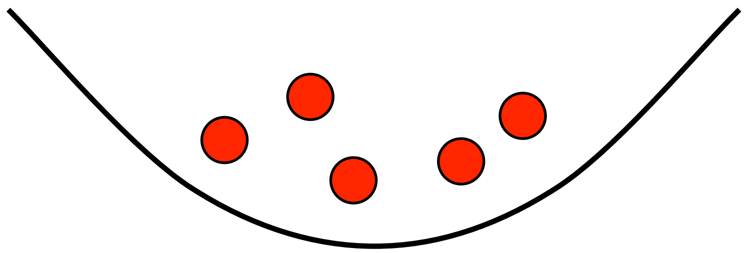
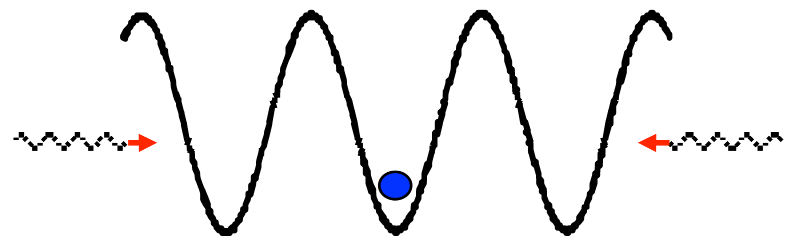
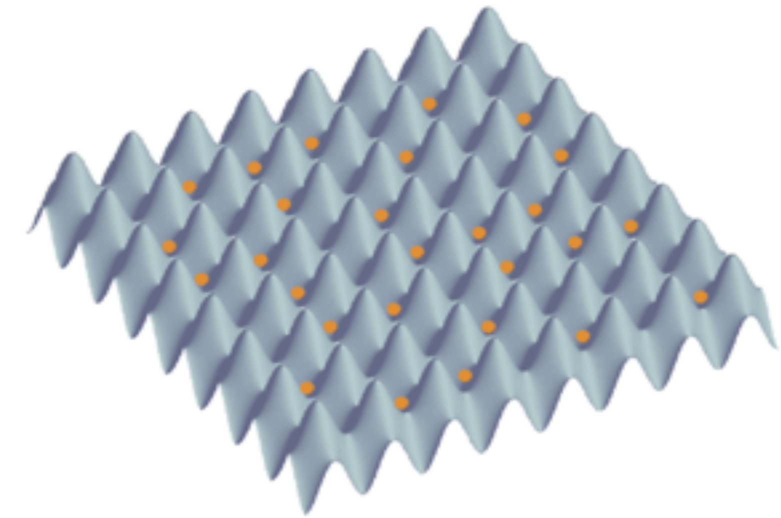
University of Strathclyde

Atoms in 3D Optical Lattices:



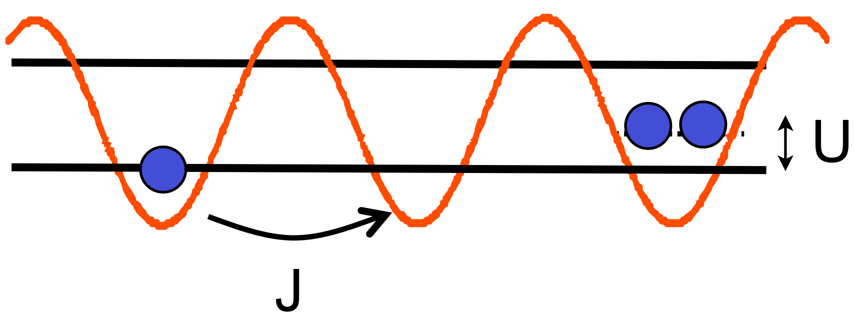




- BEC / degenerate Fermi gas

(Weakly interacting) → (Strongly interacting)

e.g., Bose-Hubbard: D. Jaksch et al. PRL ’98



$$H = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

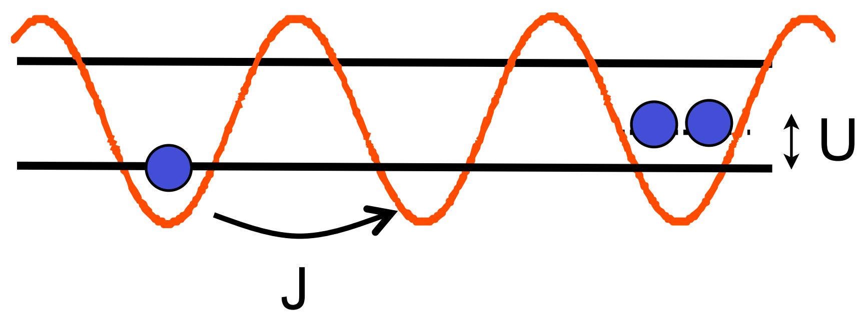
- Realise strongly correlated lattice models
- Microscopic understanding
- Study thermodynamics / quantum phases

- Discrete non-linear Schrödinger equation: Large N with small U/J, $\hat{b}_i \rightarrow b_i$

Experiments:
Munich, Zurich, NIST / JQI, MIT, Harvard, Innsbruck, Hamburg, Pisa, Florence, Oxford, Cambridge, Austin, Chicago, Penn State, Kyoto, Toronto, Stony Brook, Paris, Strathclyde, Illinois, ...

Bose-Hubbard model

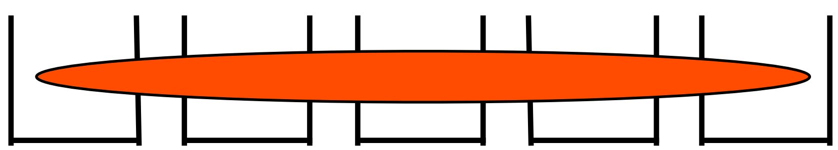
D. Jaksch et al., PRL ’98
M. Greiner et al., Nature ’02

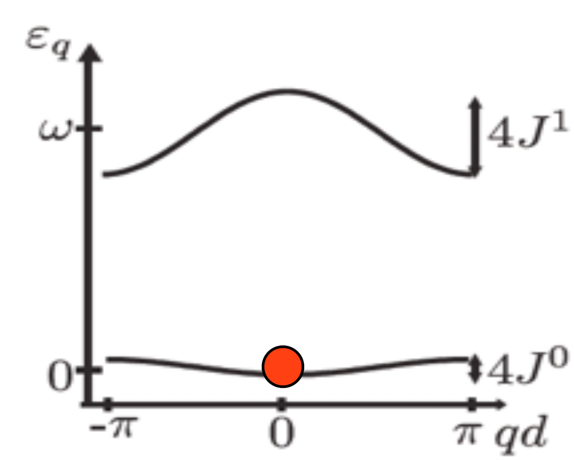


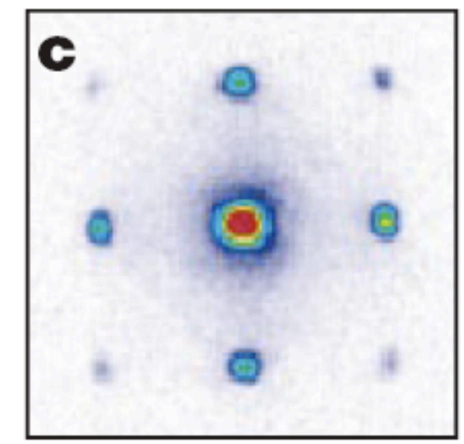
$$\hat{H} = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

Momentum Distribution

- **Superfluid** J>>U

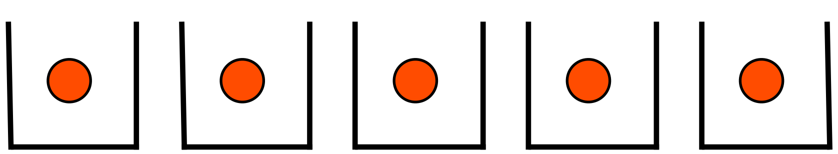


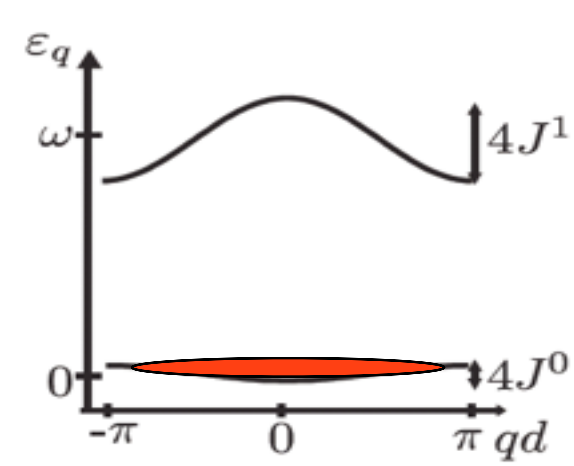


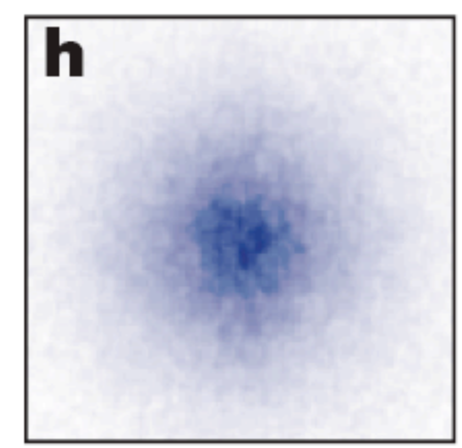


Delocalised atoms: BEC

- **Mott Insulator Phase:** J<<U





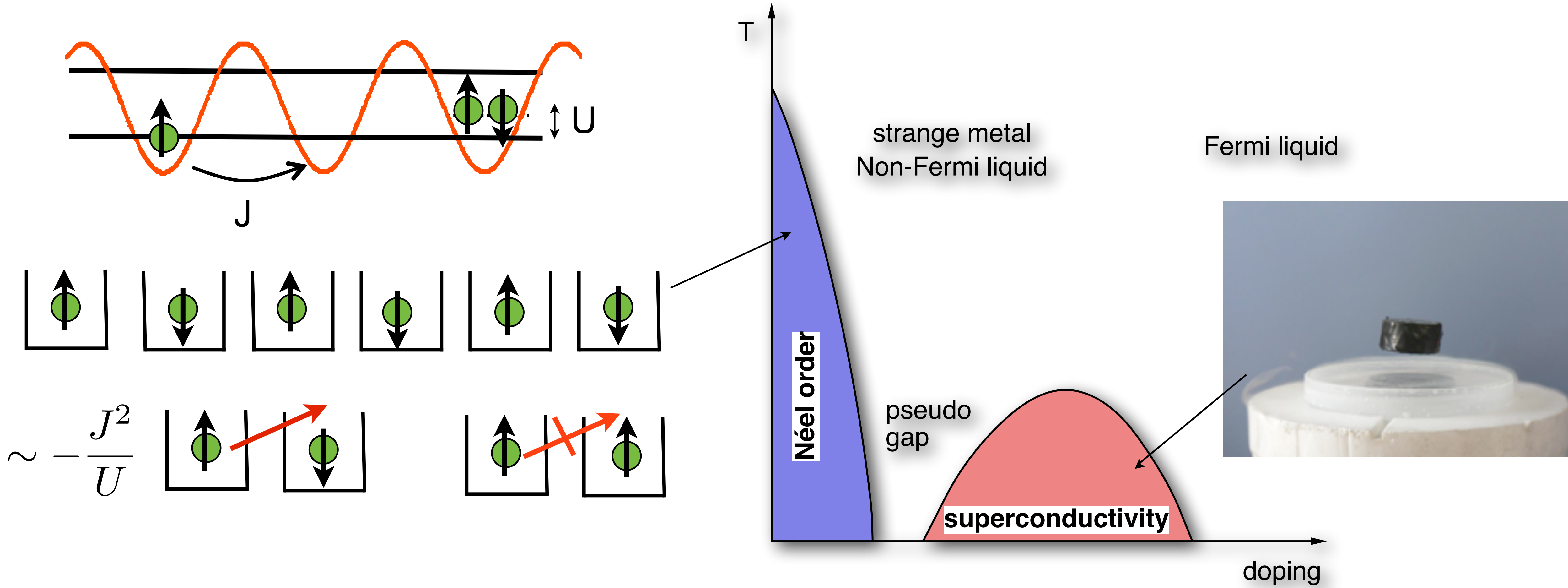


commensurate filling: atoms “pinned” by interactions

M. Greiner et al., Nature ’02

“Quantum Simulation”:

e.g., Relationship to high Tc superconductivity of cuprates - two-species experiments:



Simulations:

- Study models where we can’t access physics via classical computations
- e.g., materials engineering

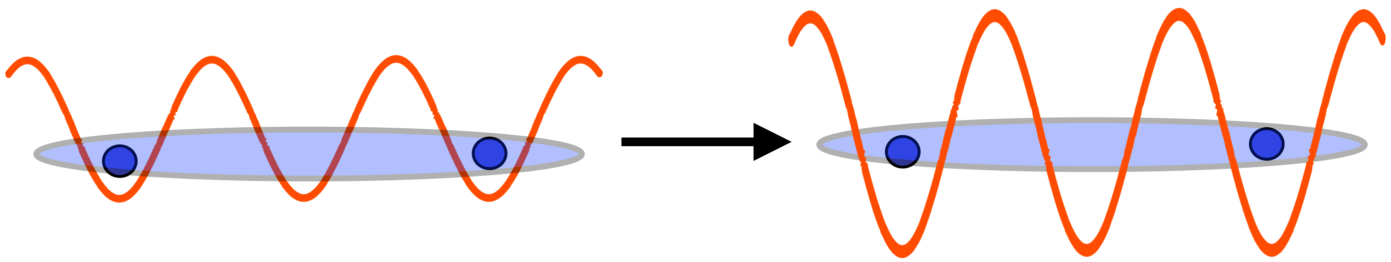
Real matter; new quantum phases

- Realise interesting many-body physics predicted but not yet observed in experiments
- Also: Exotic phases, spin models, simulators for graphene, disorder, impurities,.......

Current challenges: cooling, state preparation, control over heating in lattices

Coherent non-equilibrium dynamics:

- Intrinsic interest, e.g., Quench dynamics, thermalization, entanglement growth



M. Greiner *et al.*, Nature **419**, 51 (2002).
S. Will *et al.*, Nature **465**, 197 (2010).

M. Cheneau *et al.*, Nature **481**, 484 (2012)
J.-S. Bernier *et al.*, PRL **106**, 200601 (2011)

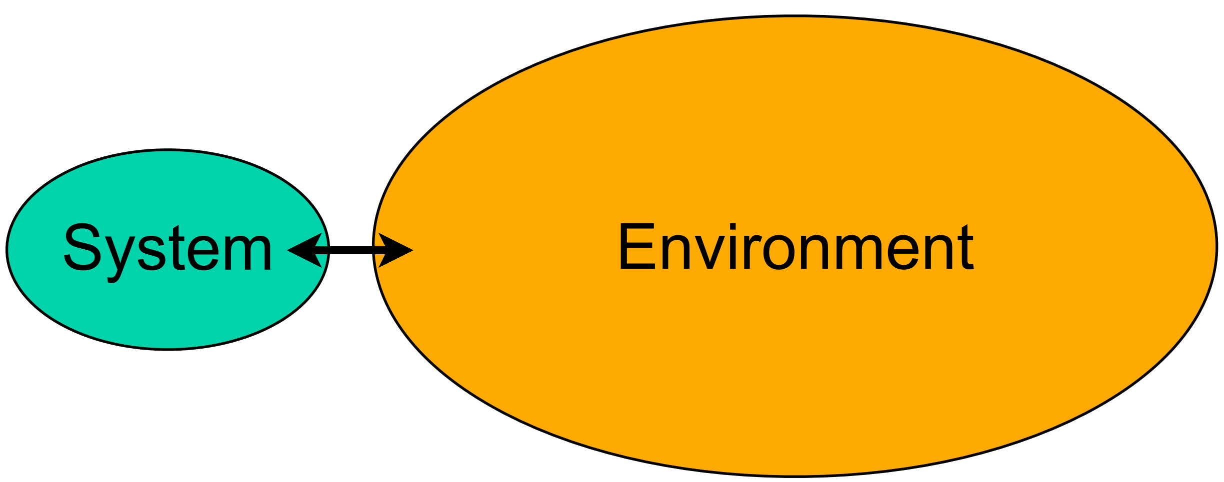
M. Rigol *et al.*, Nature **452**, 854 (2008)
M. Rigol *et al.*, PRL **98,** 050405 (2007)
M. Srednicki PRE **50**, 888 (1994)

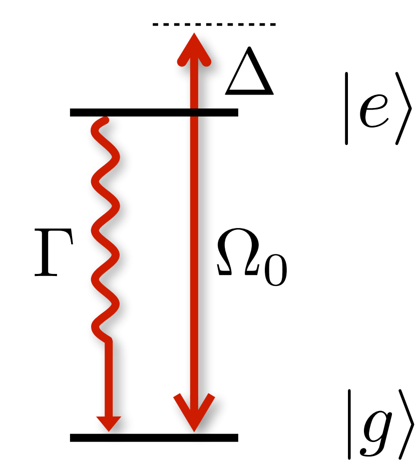
- Millisecond timescales - track+control in real time
- Long coherence times; isolated system

- Computations in 1D with time-dependent DMRG

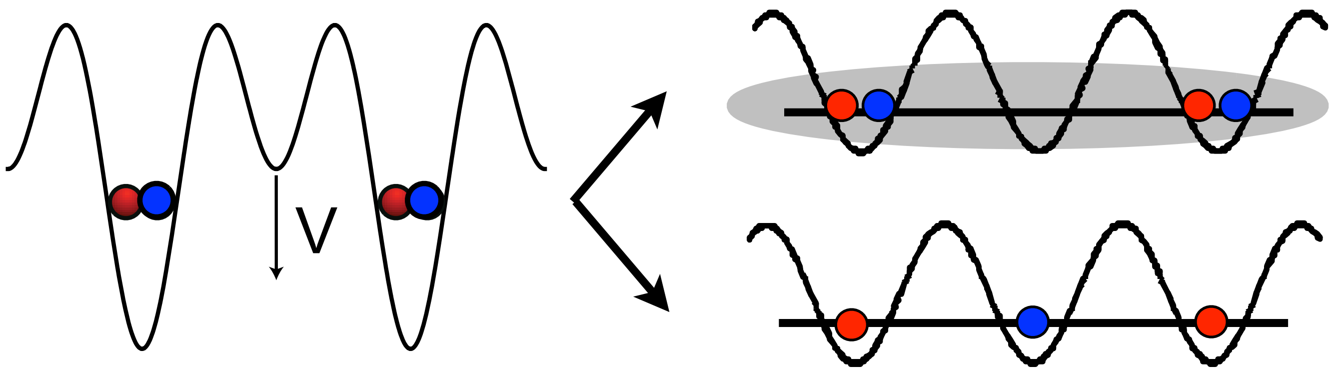
Dissipative dynamics / open many-body quantum systems:

- Understand heating / imperfections also on a microscopic level

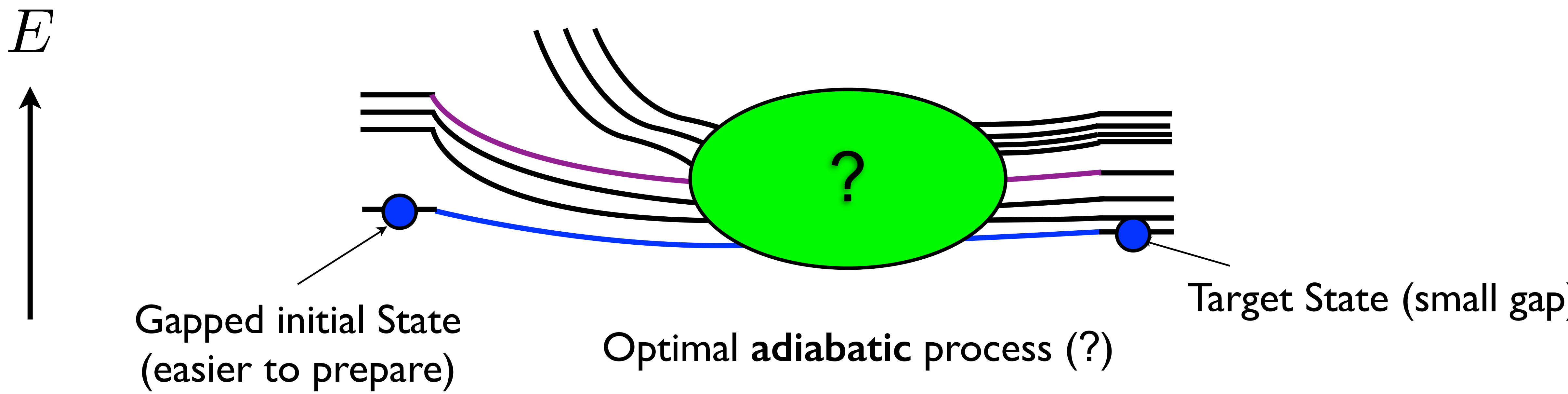




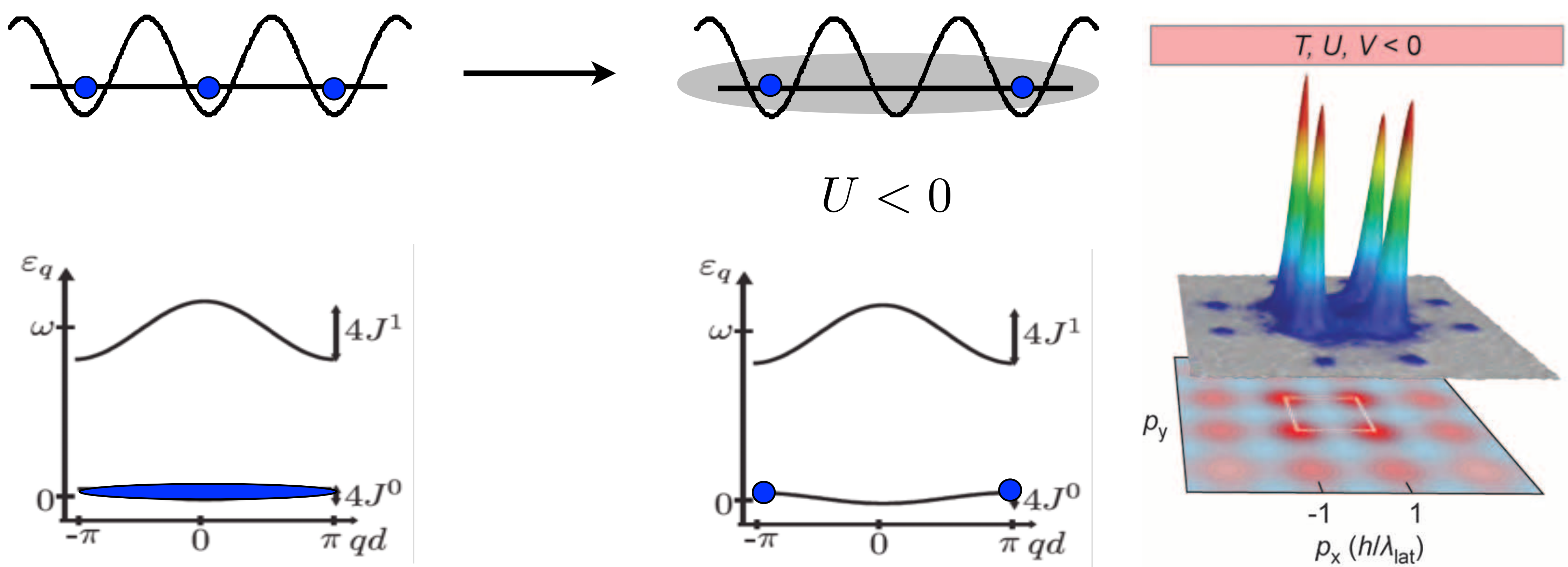
Adiabatic state preparation



Adiabatic state preparation:

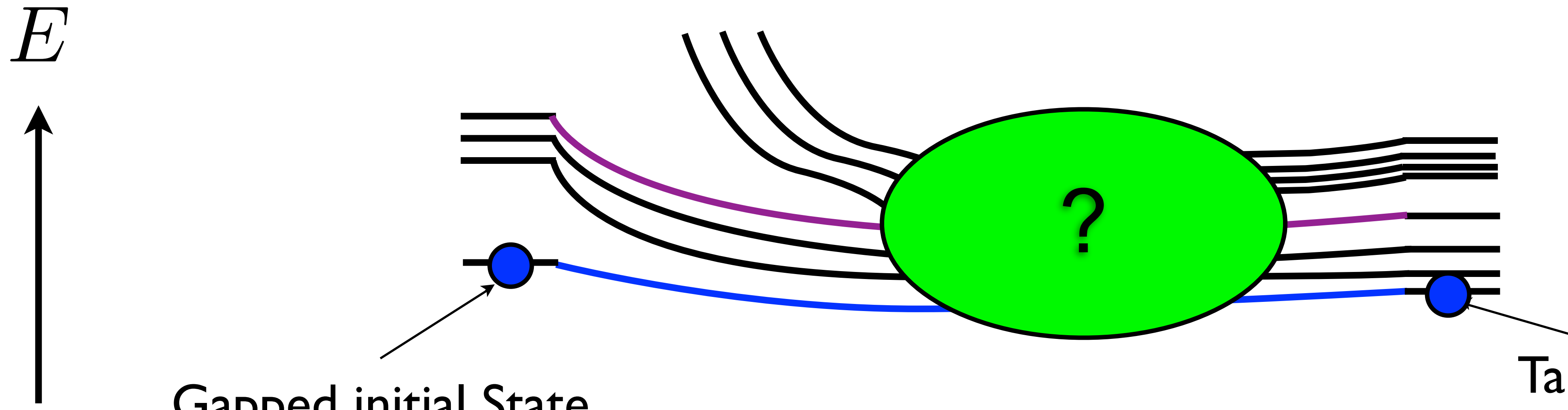


e.g.,



S. Braun *et al., Science* **339**, 52 (2013)

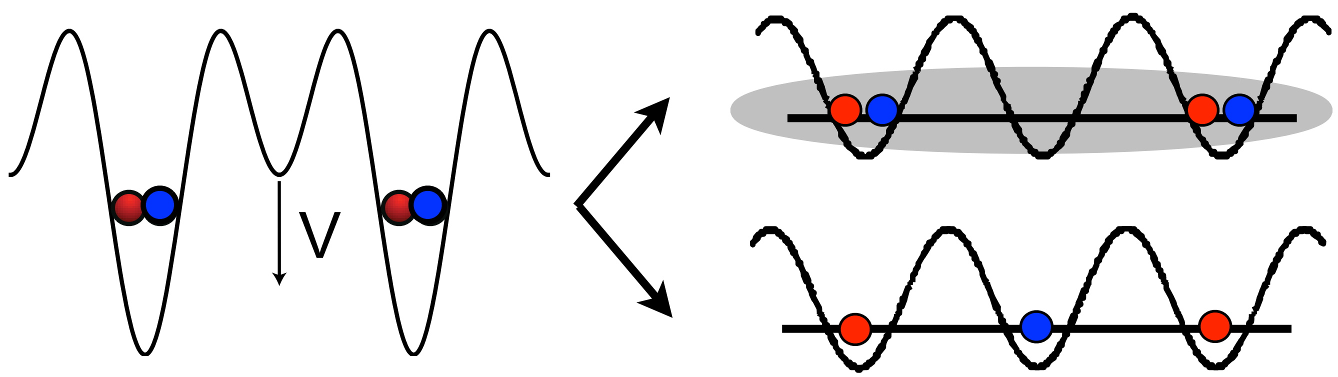
Adiabatic state preparation:



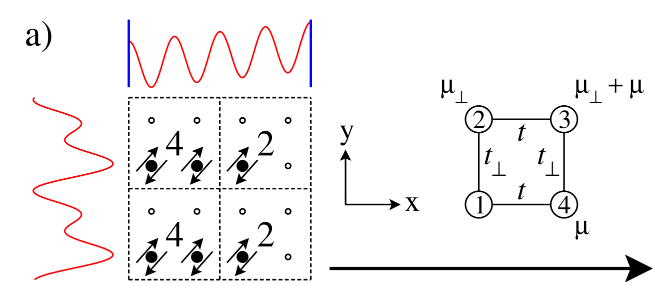
Gapped initial State
(easier to prepare)

Optimal **adiabatic** process (?)

Target State (small gap)

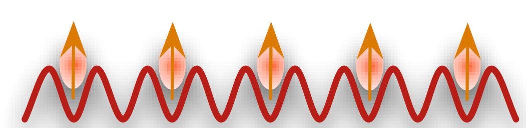


P. Rabl et al, PRL **91**, 110403 (2003)
A. Kantian et al., PRL **104**, 240406 (2010)
M. Lubasch et al., PRL **107**, 165301 (2011)

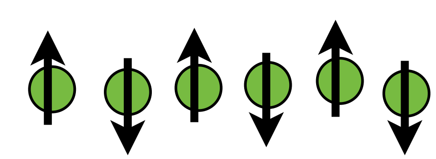


S. Trebst et al., PRL **96**, 250402 (2006)

T.-L. Ho and Q. Zhou, PRL **99**, 120404 (2007)

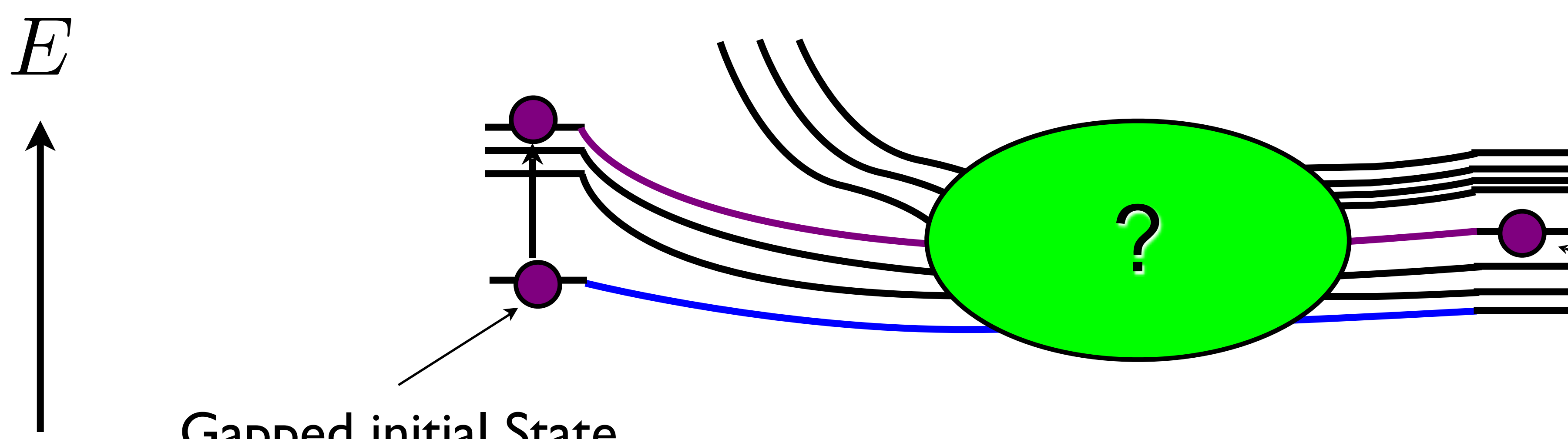


J. Schachenmayer et al., New J. Phys. **12**, 103044 (2010)
T. Pohl, E. Demler, M. D. Lukin PRL **104** 043002 (2010)



A. M. Rey et al., PRL **99**, 140601 (2007)
A. S. Sørensen et al., PRA **81**, 061603(R) (2010)

Adiabatic state preparation:



Non-adiabatic + adiabatic process

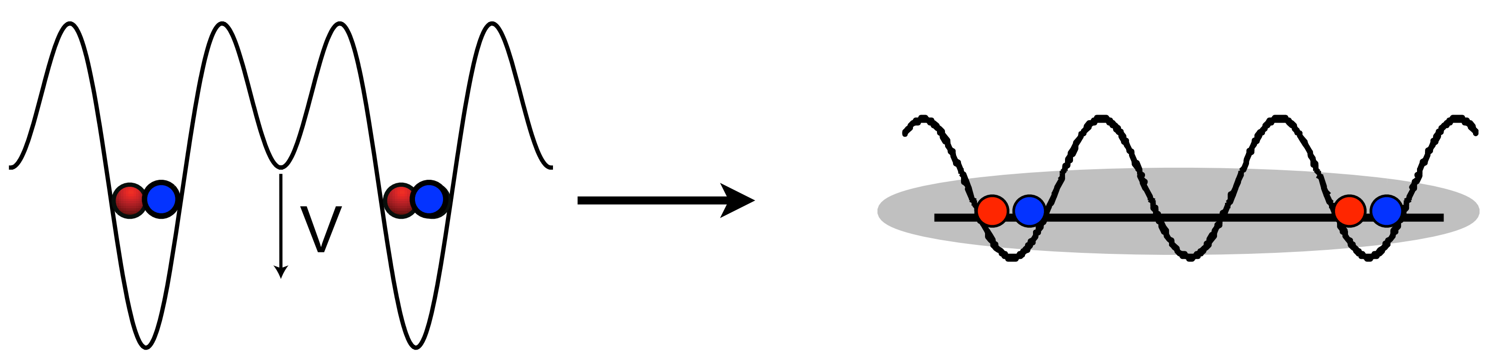
Exact excited eigenstates of the Hubbard model for fermions: η pairs

- C. N. Yang PRL (1989):

$$H = -J \sum_{\langle i,j \rangle,\sigma} c^\dagger_{i,\sigma} c_{j,\sigma} + U \sum_i c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow} c_{i,\downarrow} c_{i,\uparrow} \qquad \eta^\dagger \sim \sum_i (-1)^i c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow}$$

$$[H, \eta^\dagger] = U\eta^\dagger \qquad H\left(\eta^\dagger\right)^N |\mathrm{vac}\rangle = NU\left(\eta^\dagger\right)^N |\mathrm{vac}\rangle$$

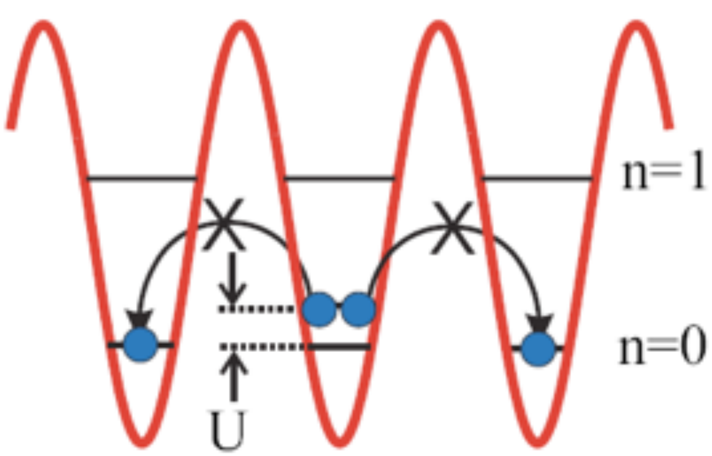
- A. Kantian, A. J. Daley, and P. Zoller, PRL 104, 240406 (2010):



Repulsively bound atom pairs

K. Winkler et al., Nature **441**, 853 (2006)

- Weak dissipative processes - metastable many-body states can be prepared and studied in an experiment

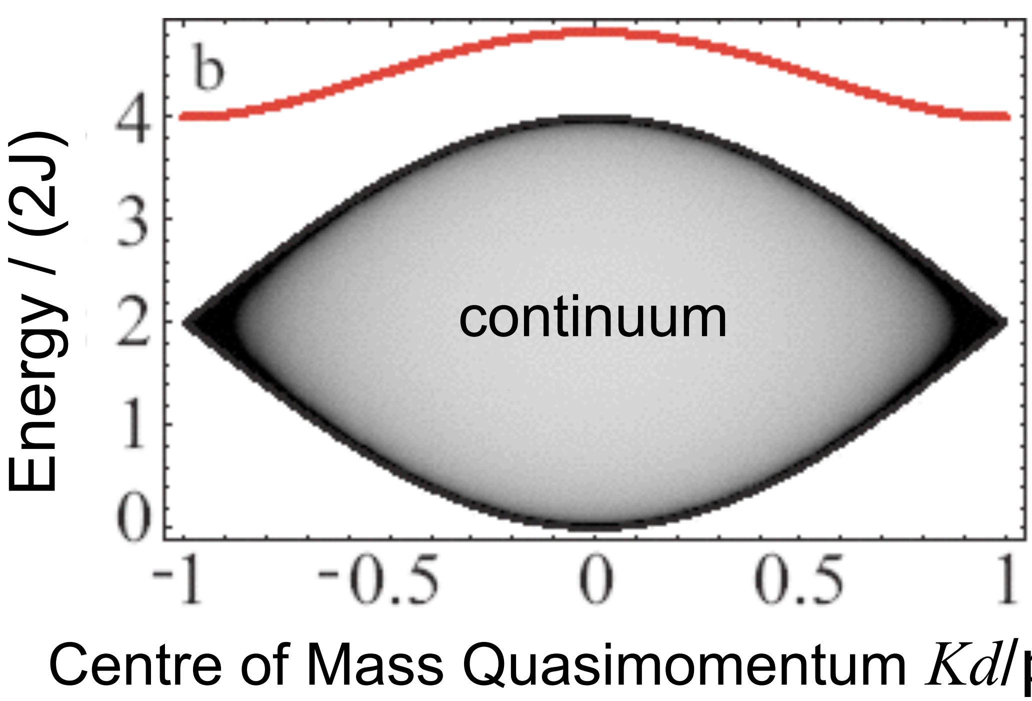


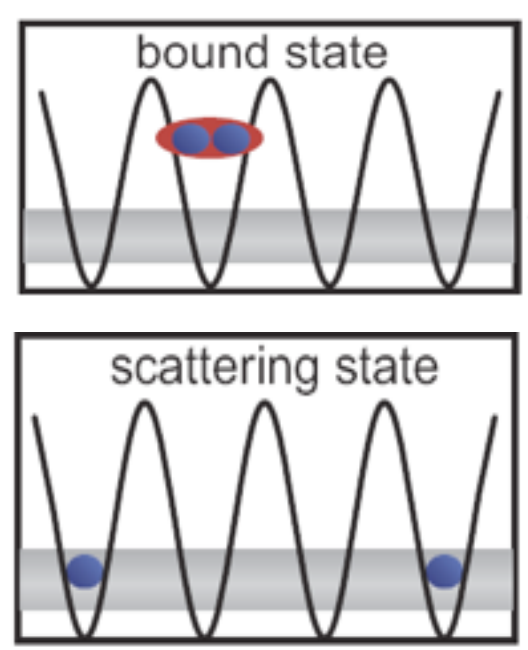
Repulsively bound bosonic pairs for large U/J:

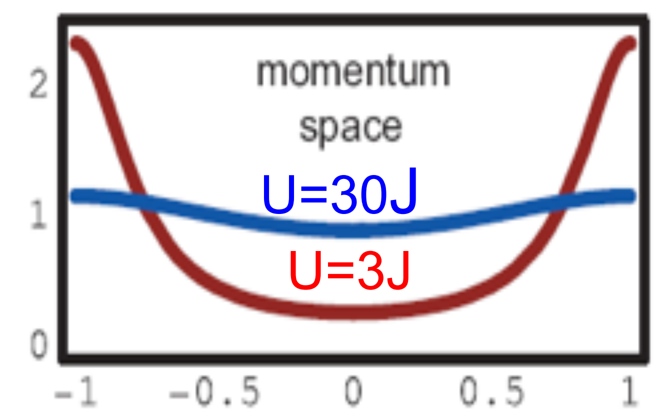
- Pairs are stable (cannot convert large repulsive "binding energy" into kinetic energy J
- No dissipative decay channels
- Composite object tunnelling J^2/U

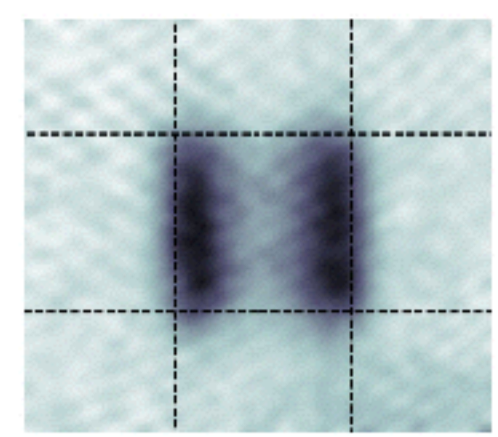
Quasi- momentum distribution

Two particles on a lattice:







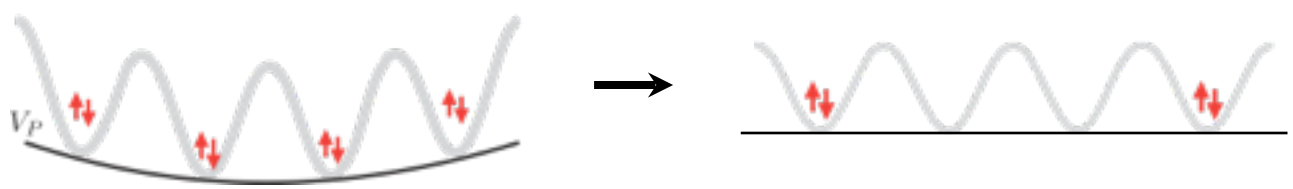


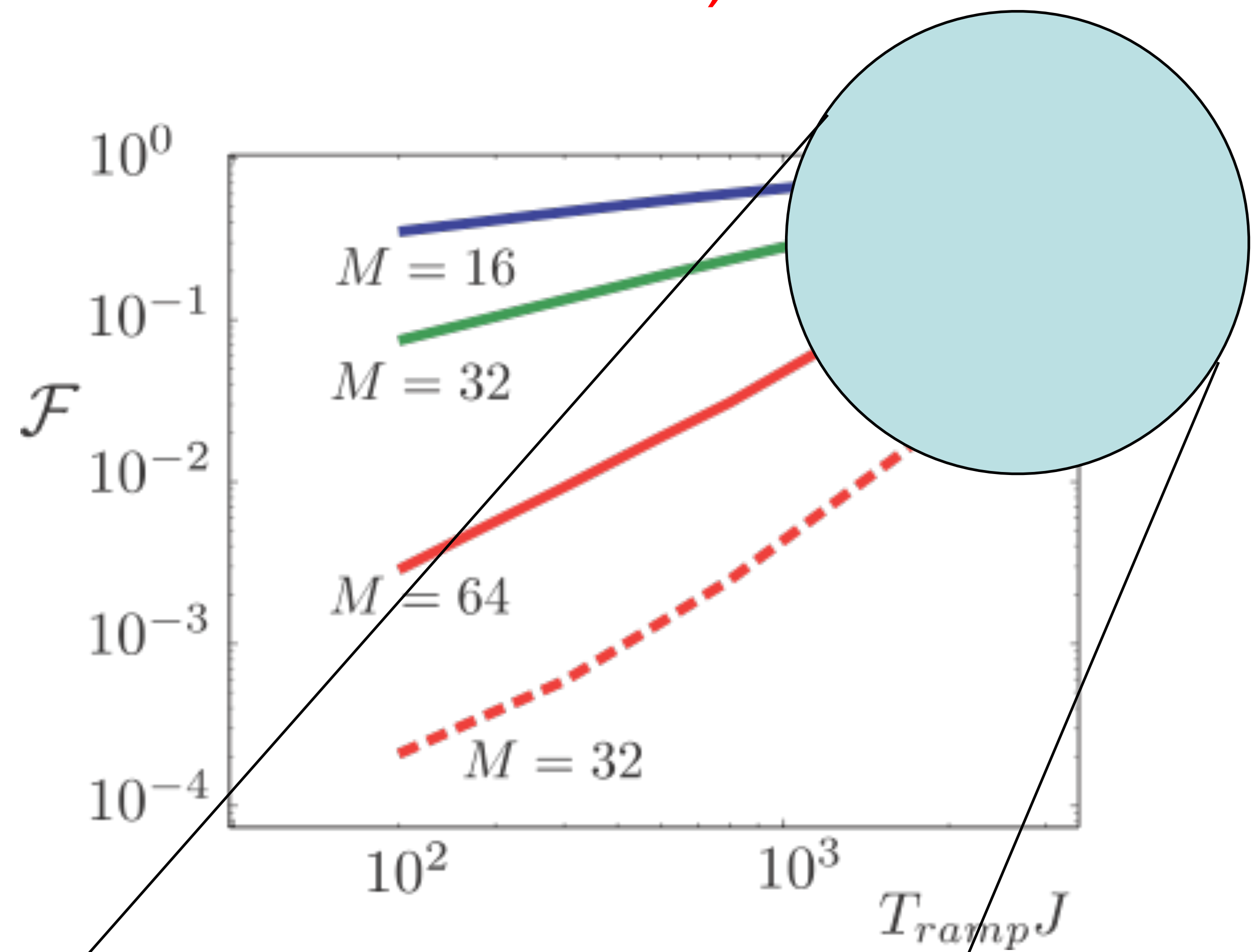
Preparation of an eta condensate (t-DMRG simulation results):

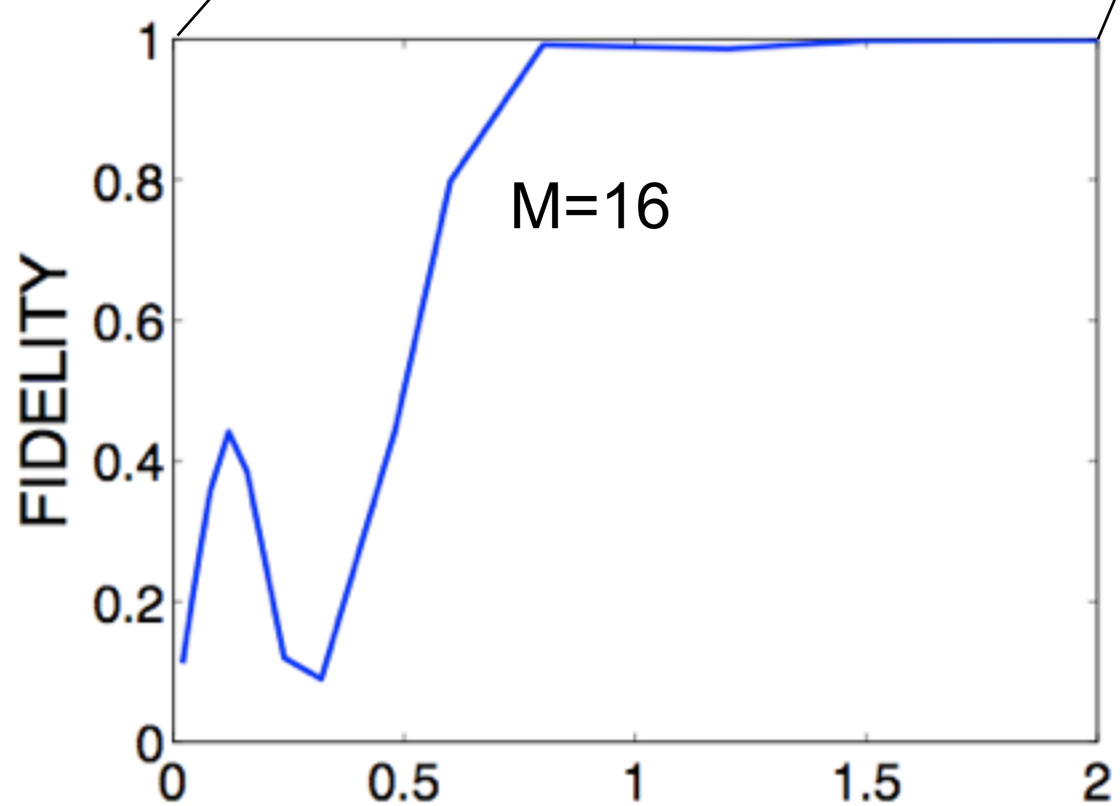
Many-body state fidelity: $\mathcal{F} = |\langle \psi_f | \psi_N \rangle|^2$

- Very sensitive measure for large systems
- Obtain ~100% state fidelity for long ramps

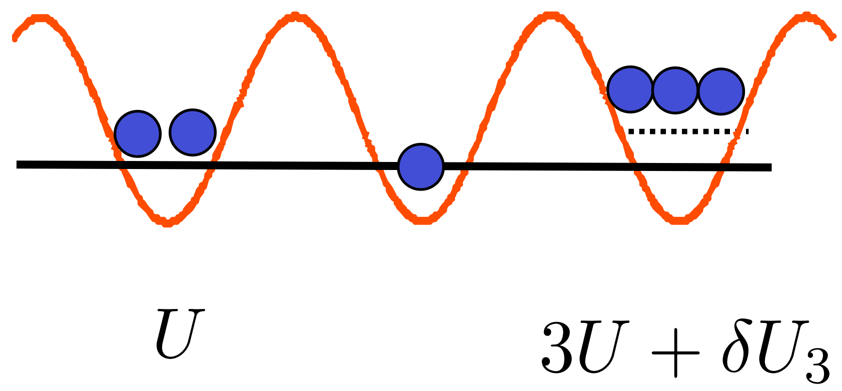
- Dashed line: opening a harmonic trap
  A. Rosch et.al., PRL 101, 265301 (2008)







A. Kantian, A. J. Daley, and P. Zoller, PRL 104, 240406 (2010)



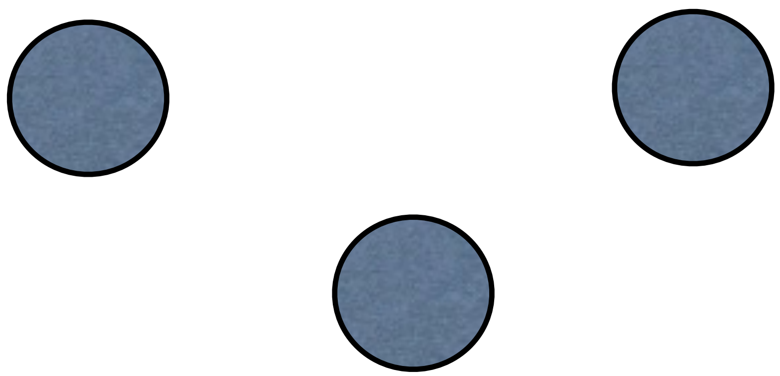
Three-body interactions

$$H_{\text{eff}} = \frac{U_{\text{eff}}^{(2)}}{2} \sum_i n_i(n_i - 1) + \frac{U_{\text{eff}}^{(3)}}{6} \sum_i n_i(n_i - 1)(n_i - 2)$$

- Three-body constraints
- DNLSE: Nonlinearity $|\psi|^2 + \lambda|\psi^4|$

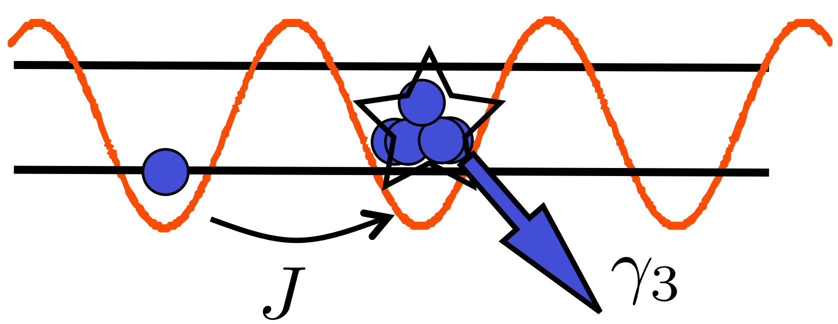
3-body loss as a dynamical 3-body interaction

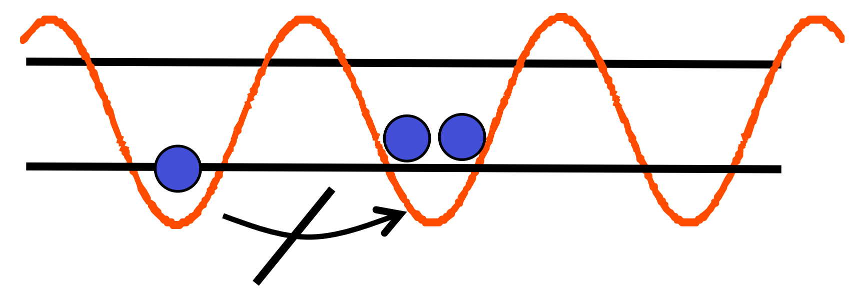
3-body loss processes (-):



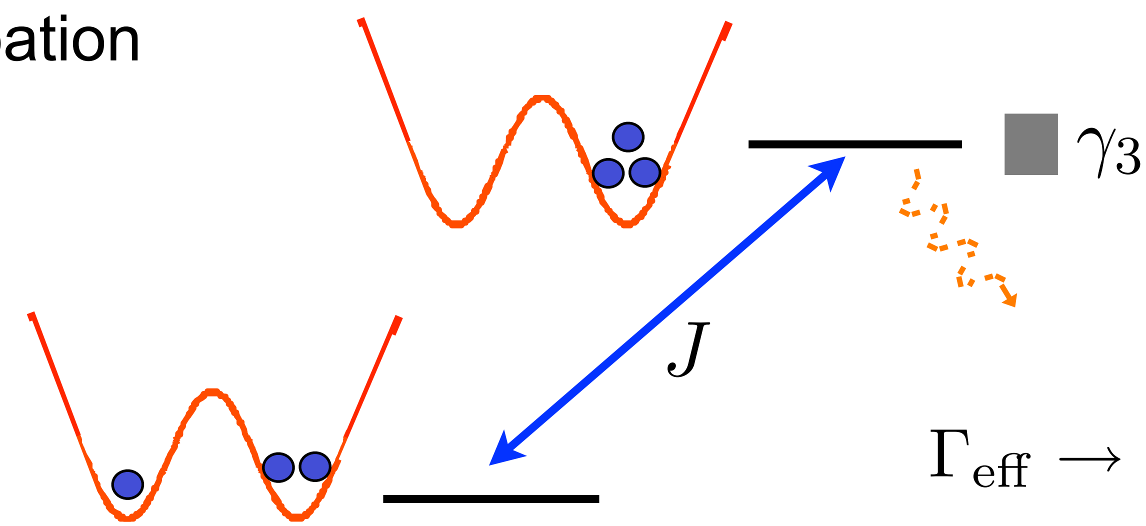
- Three atom collision
- Molecule and atoms ejected from lattice
- Ubiquitous / typically undesirable in cold atoms

3-body interactions (+):





- Dynamical suppression of 3-body occupation
  - cf. broadened resonance



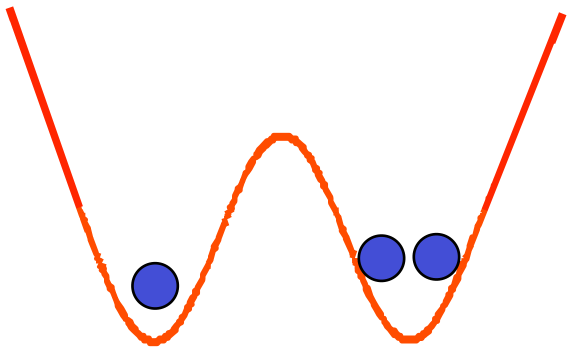
$$\Gamma_{\text{eff}} \rightarrow \frac{J^2}{\gamma_3}$$

(cf. "Quantum Zeno effect")

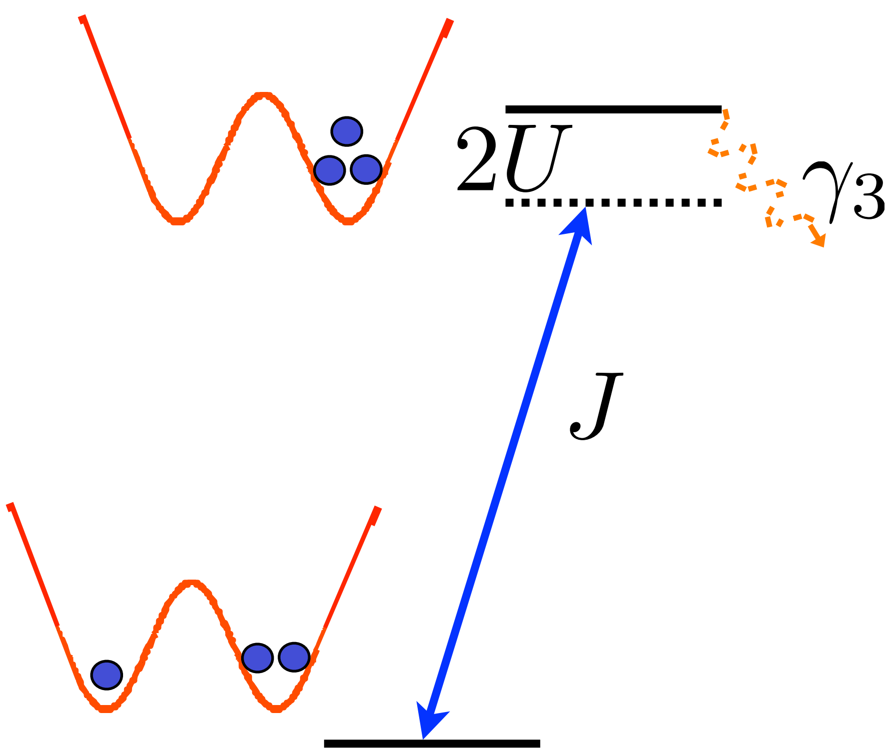
W.M. Itano, D.J. Heinzen, J.J. Bollinger, and D.J. Wineland, PRA 41, 2295 (1990)

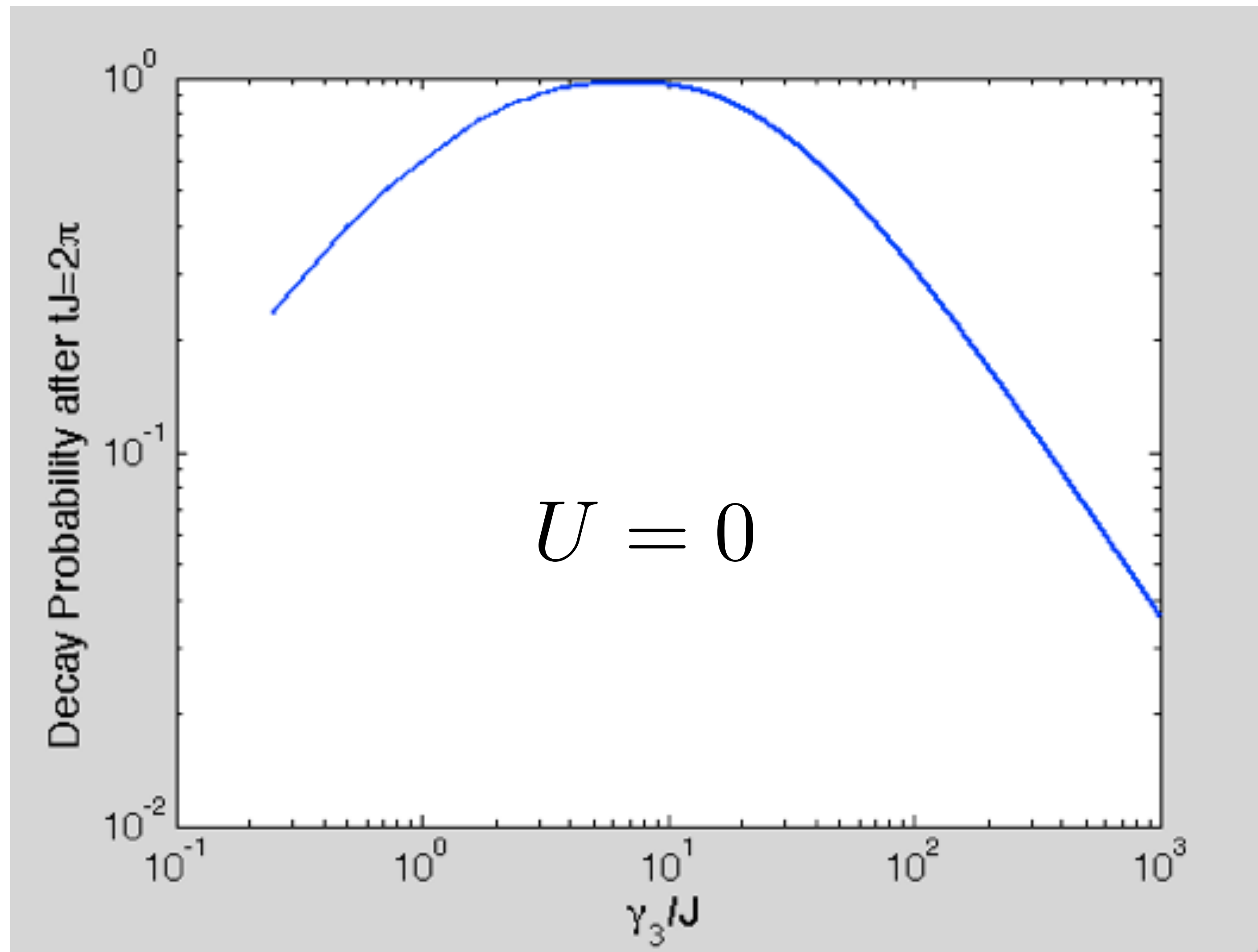
Example: Atoms in a double-well

Effective loss rate: $\gamma_3 \gg U, J \qquad \Gamma_{\mathrm{eff}} \approx \frac{6\,J^2}{\gamma_3}$



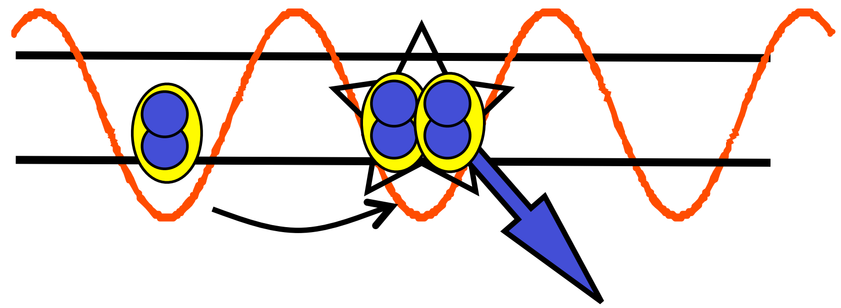
Large on-site 3-body loss rate leads to suppression of 3-atom occupation

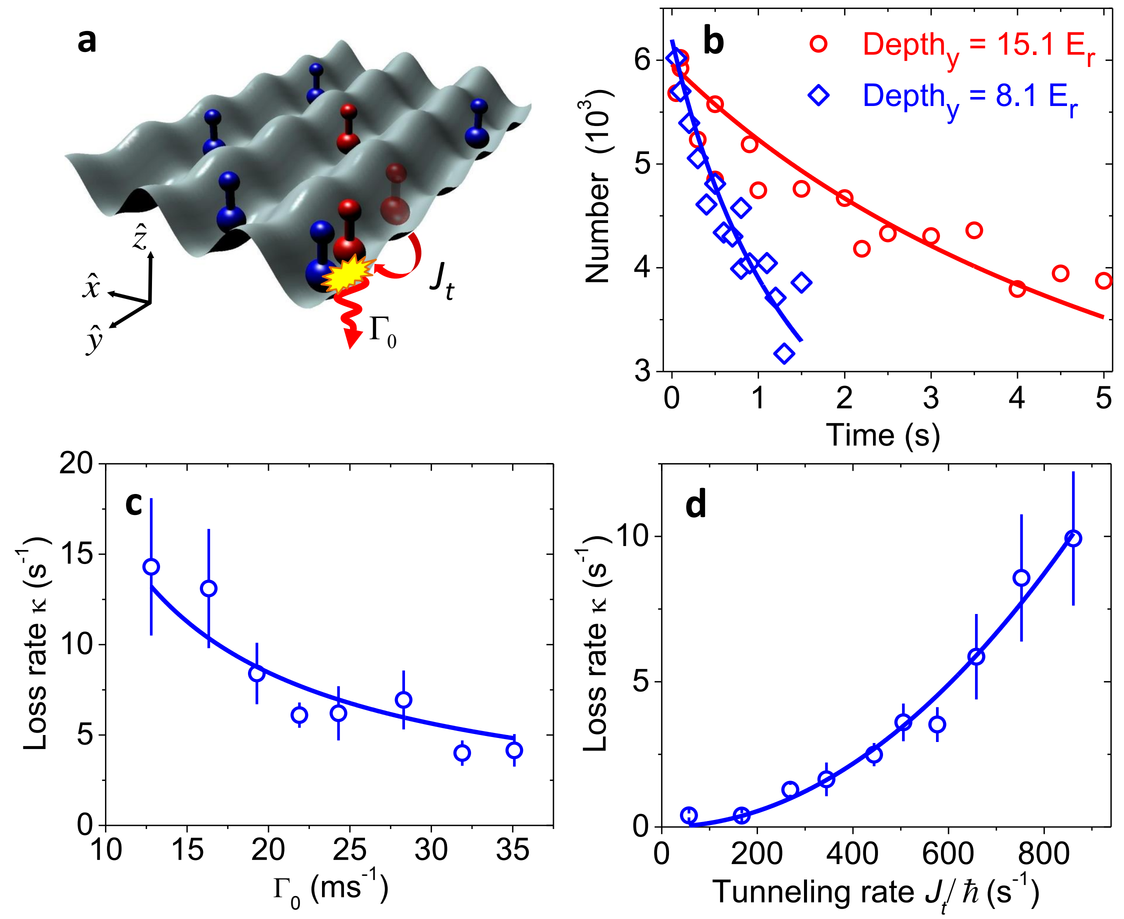




Master Equation: $$\dot{\rho} = -i[H,\rho] - \frac{\gamma_3}{12}\sum_i \left[(c_i^\dagger)^3 c_i^3 \rho + \rho (c_i^\dagger)^3 c_i^3 - 2 c_i^3 \rho (c_i^\dagger)^3 \right]$$

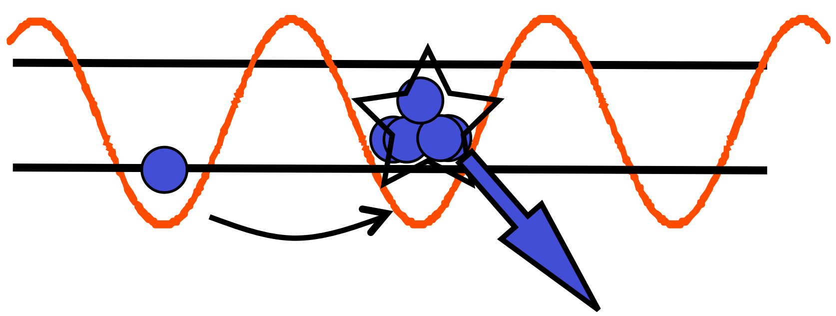
Effective 2-body interactions:

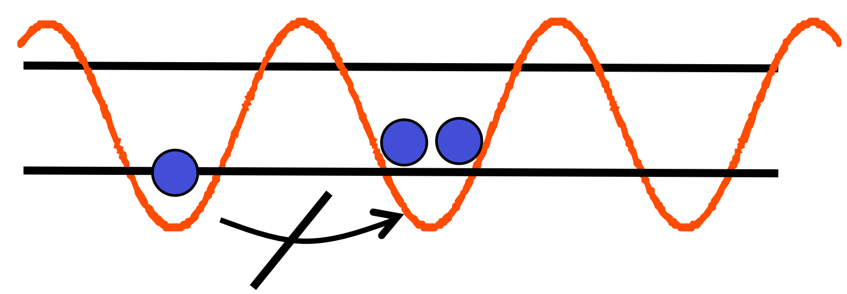




- Observation for loss of molecules
  N. Syassen et al., Science 320, 1329 (2008)
  B. Yan et al., Nature 501, 521 (2013)
  B. Zhu et al., PRL 112, 070404 (2013)
  J. J. Garcia-Ripoll et al., NJP. ***11****, 013053 (2009)*

Effective 3-body interactions:

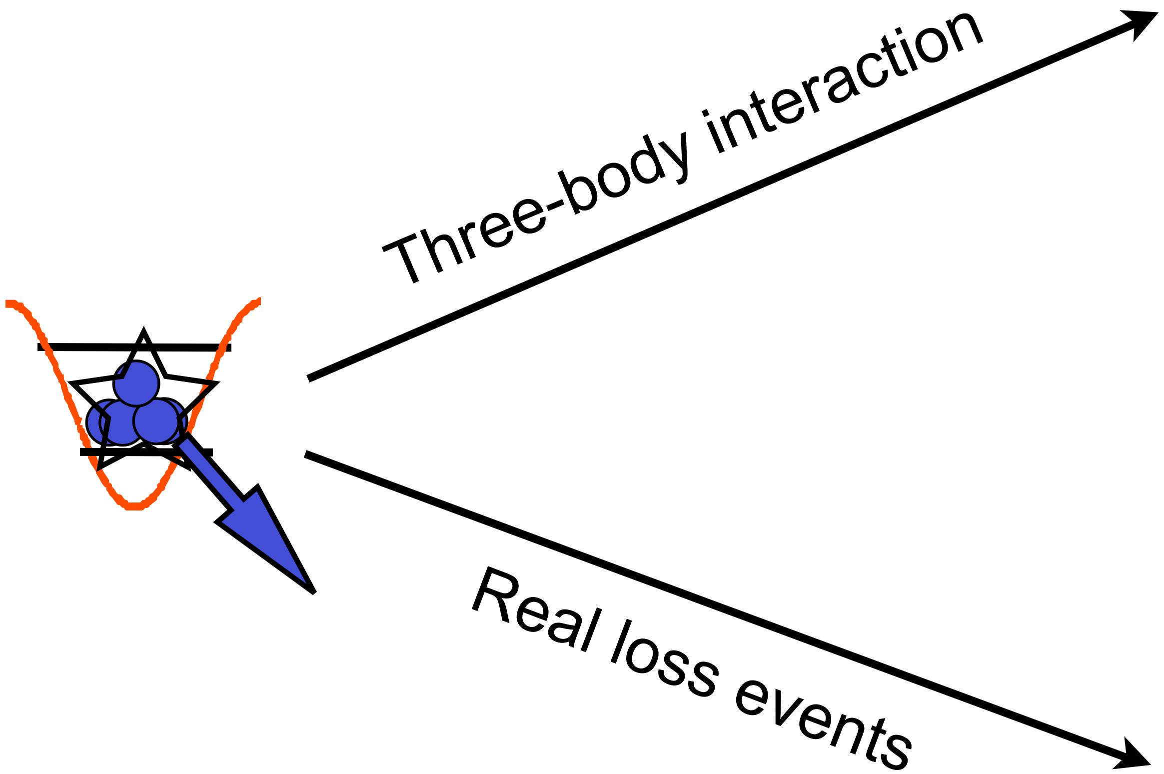




State engineering:

- Bosons: Stabilise system for attractive interactions
- Fermions: Stabilise 3-species mixture - atomic colour superfluid

(Related work on rotating continuous systems, M Roncaglia, M Rizzi, J. I Cirac, Phys. Rev. Lett. 104, 096803 (2010))



Physics including hard-core constraint

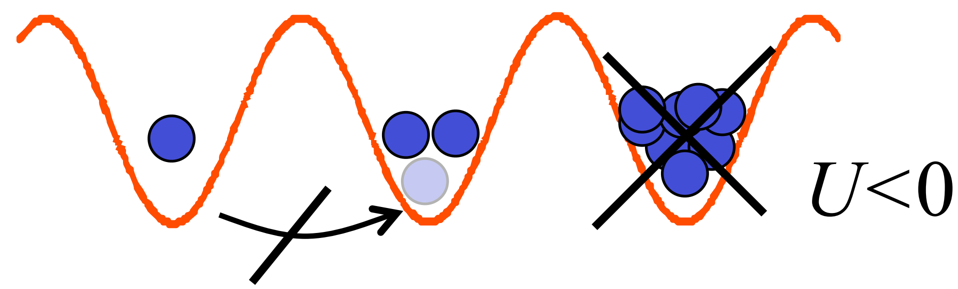
- Assume loss suppression is perfect
- No three-body occupation
- Static / Ground-state properties

Time-dependent non-equilibrium dynamics

- Test how well suppression works
- Study dynamics when loss occurs
- Many-body numerical simulations of master equation

A. J. Daley, J. M. Taylor, S. Diehl, M. Baranov, and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009)

Physics arising from hard-core three-body interaction:



Projected Bose-Hubbard model:

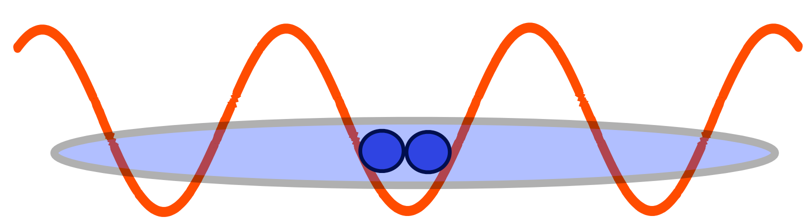
- Suppression of onsite occupation > 2
- System with attractive 2-body U/J<0 now stable

$$H = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

$$(b_i^\dagger)^3 \equiv 0$$

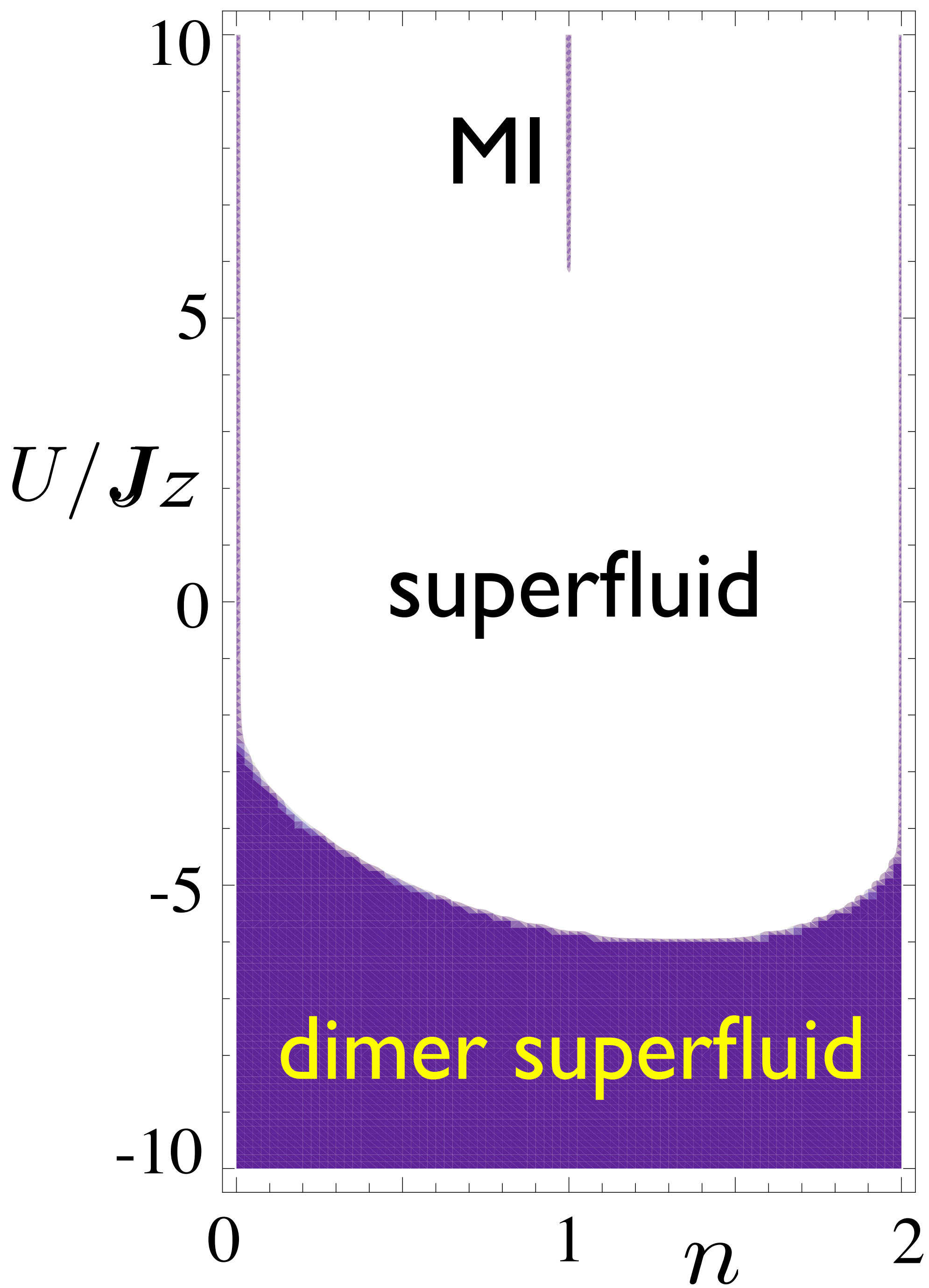
New Feature:

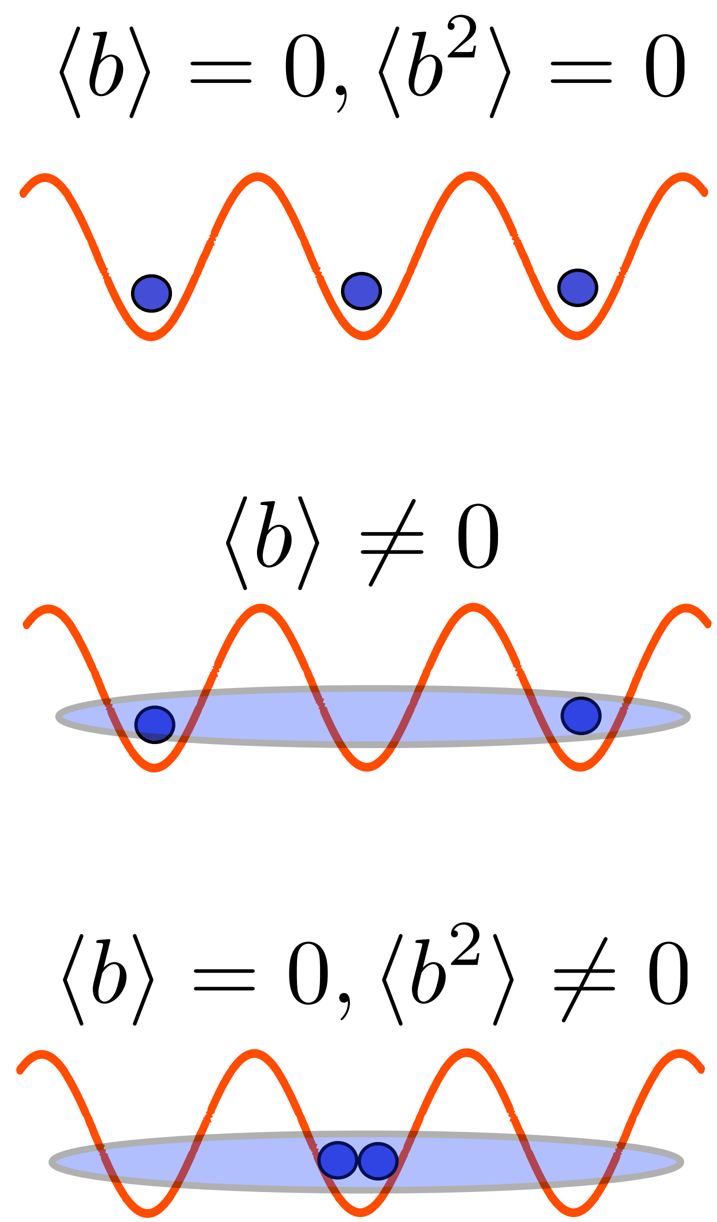
- Dimer superfluid phase



A. J. Daley, J. M. Taylor, S. Diehl, M. Baranov, and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009).
S. Diehl, M. Baranov, A. J. Daley, and P. Zoller, Phys. Rev. Lett. **104**, 165301 (2010)
L. Bonnes and S. Wessel, Phys. Rev. Lett. **106**, 185302 (2011)
Kwai-Kong Ng and Min-Fong Yang, Phys. Rev. B **83**, 100511(R) (2011)

Qualitative picture - mean-field phase diagram:



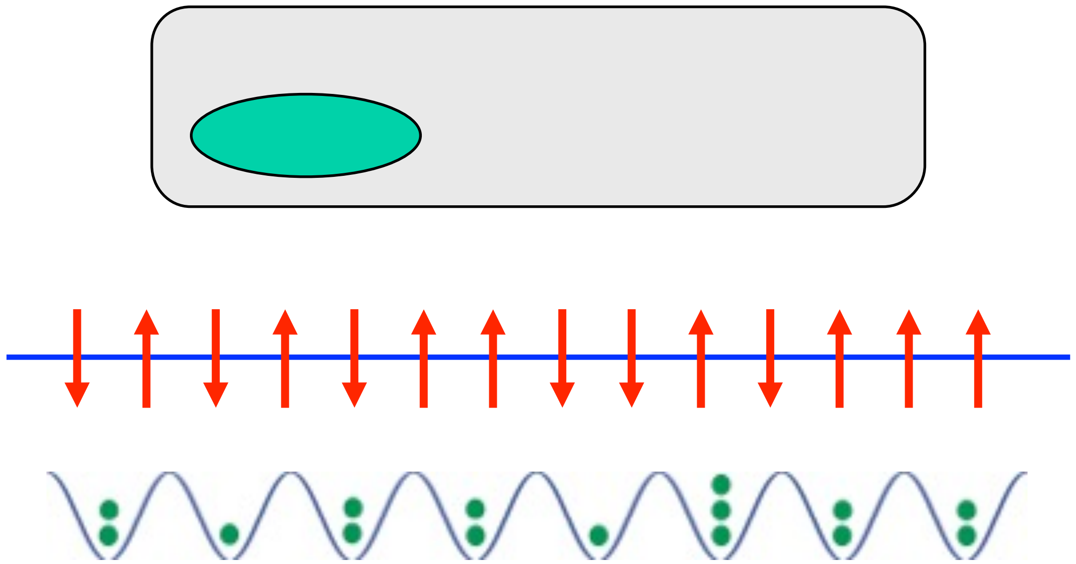


A. J. Daley, J. M. Taylor, S. Diehl, M. Baranov, and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009).
S. Diehl, M. Baranov, A. J. Daley, and P. Zoller, Phys. Rev. Lett. **104**, 165301 (2010)
L. Bonnes and S. Wessel, Phys. Rev. Lett. **106**, 185302 (2011)
Kwai-Kong Ng and Min-Fong Yang, Phys. Rev. B **83**, 100511(R) (2011)

Time-dependent dynamics in 1D: t-DMRG + Quantum Trajectories

t-DMRG Algorithm

- Integration of many-body Schrödinger eq. in 1D on lattice/spin Hilbert space.



- Works in 1D, near equilibrium
- Compute ground states / time evolution
- Direct determination of dynamics for typical experimental parameters

G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003)
G. Vidal, Phys. Rev. Lett. **93**, 040502 (2004)
A. J. Daley, C. Kollath, U. Schollwöck, and G.Vidal, Journal Stat. Mech.: Th and Exp. P04005 (2004)
S. R. White and A. E. Feiguin, PRL **93**, 076401 (2004)
F. Verstraete, V. Murg, and J. I. Cirac, Advances in Physics 57, 143 (2008).

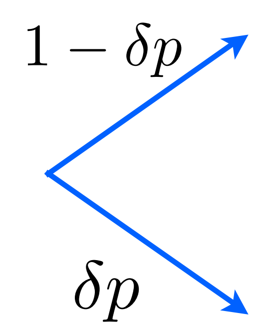
Quantum Trajectories

$$\dot{\rho} = -i[H,\rho] - \frac{\Gamma}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

- Developed to compute dynamics under master equations

H. Carmichael, *An Open Systems Approach to Quantum Optics*
K. Mølmer, J. Dalibard, Y. Castin, JOSA B **10**, 524 (1993)
R. Dum *et al.*, PRA **46**, 4382 (1992)

- Evolve stochastic trajectories (states) with two possible operations per timestep:



- Evolution under

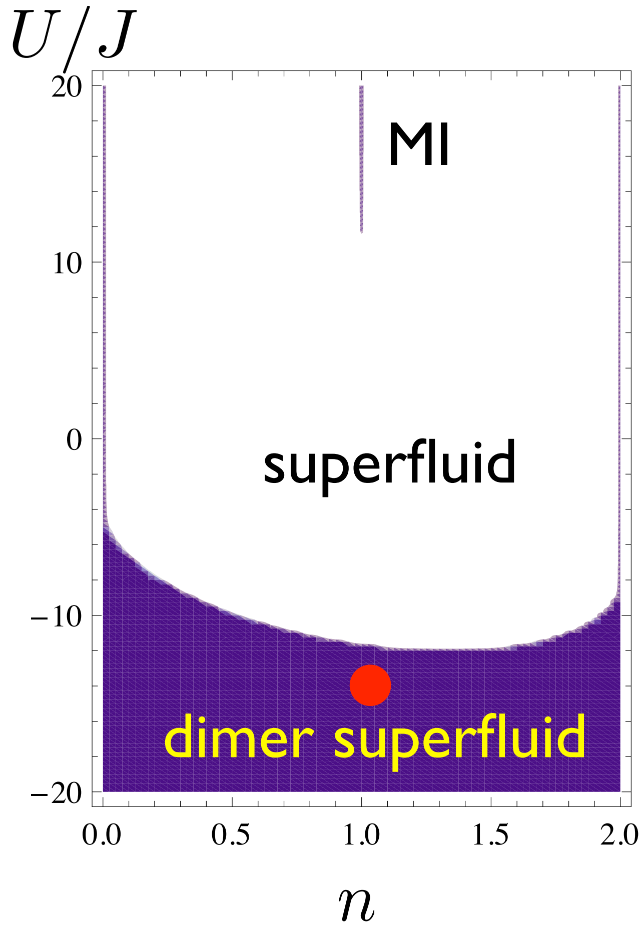
$$H_{\text{eff}} = H - i\frac{\Gamma}{2}\sum_m c_m^\dagger c_m$$

- or Quantum Jumps

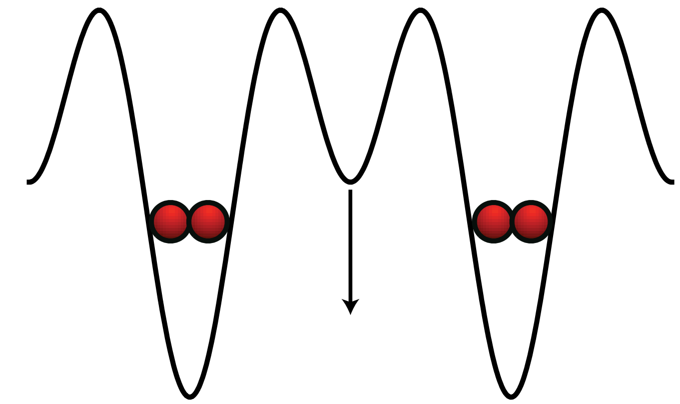
$$|\psi\rangle = \frac{c_m|\psi\rangle}{||c_m|\psi\rangle||}$$

- Expectation values by stochastic average.
- Trade-off: Smaller local Hilbert space vs. trajectory averages

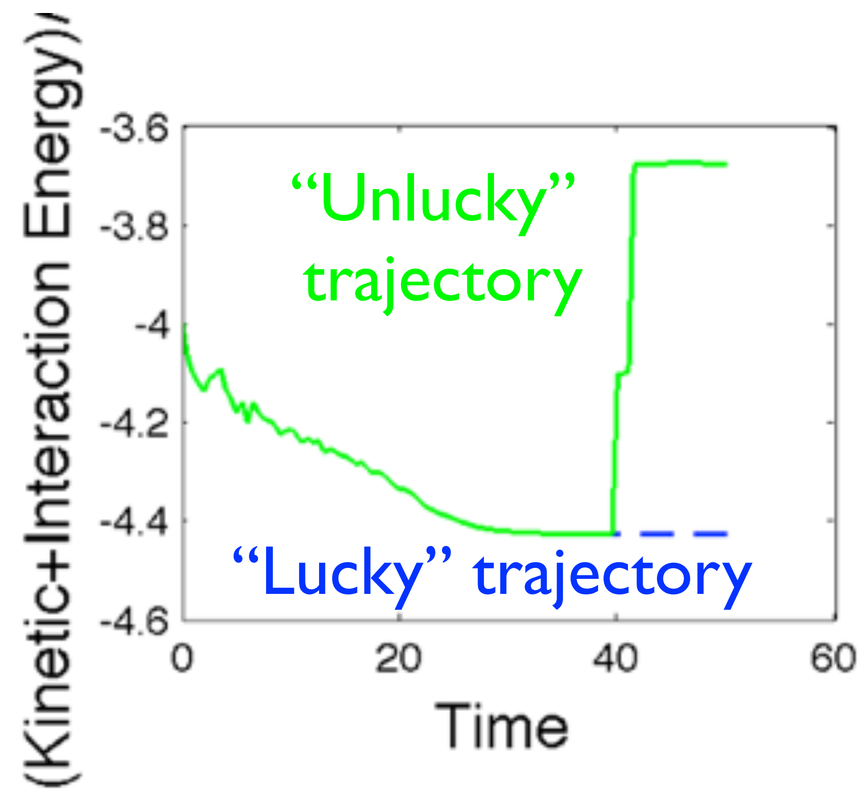
A. J. Daley et al., Phys. Rev. Lett **102**, 040402 (2009).

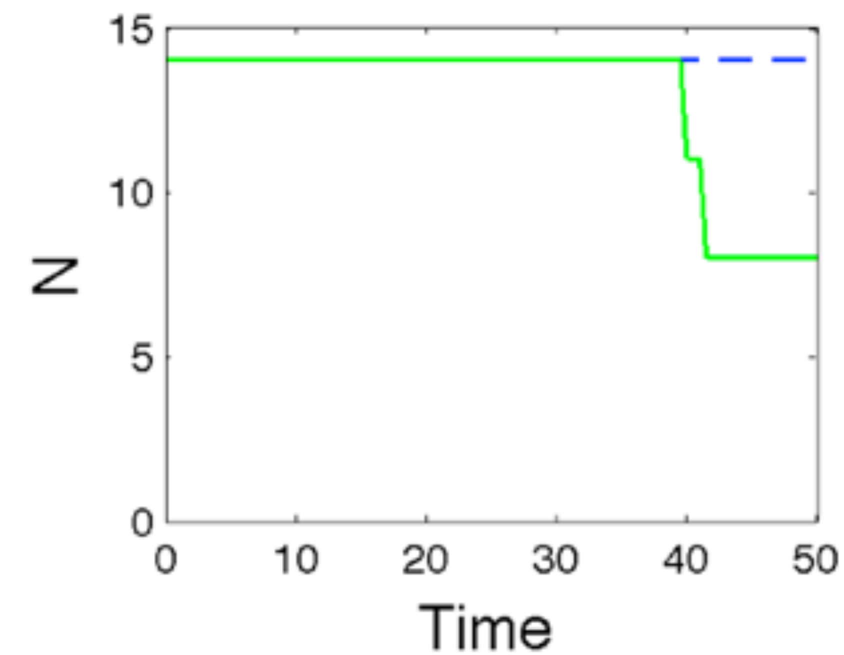


Ramp: Superlattice, V/J=30 to V/J=0,
N=M=20
Quantum trajectories + t-DMRG



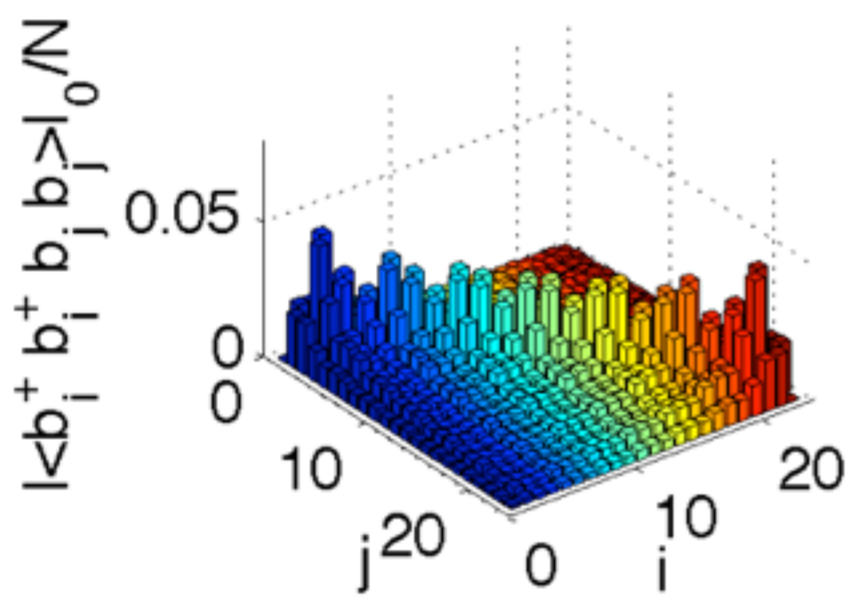
$$\Gamma = 250J$$

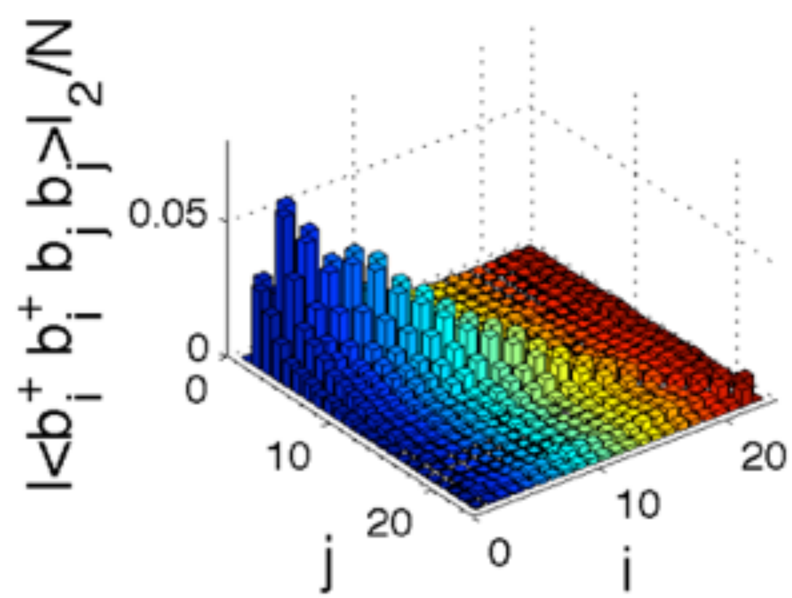




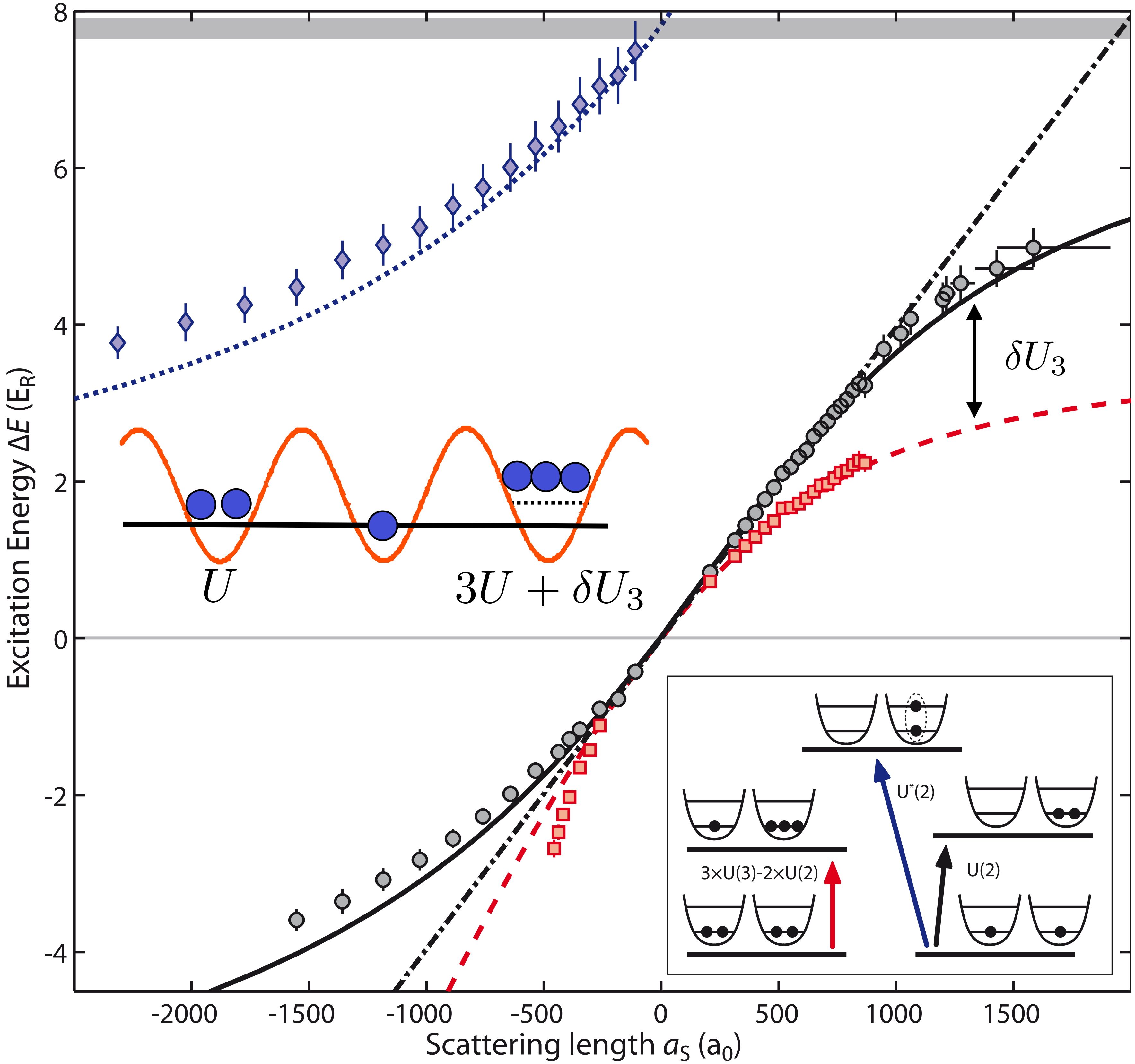
"Unlucky" trajectory

"Lucky" trajectory





What about using coherent shifts?



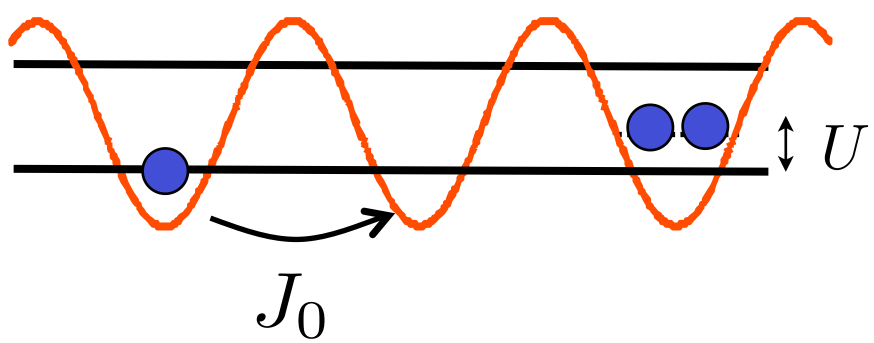
M. Mark et al., Phys. Rev. Lett. **108**, 215302 (2012)

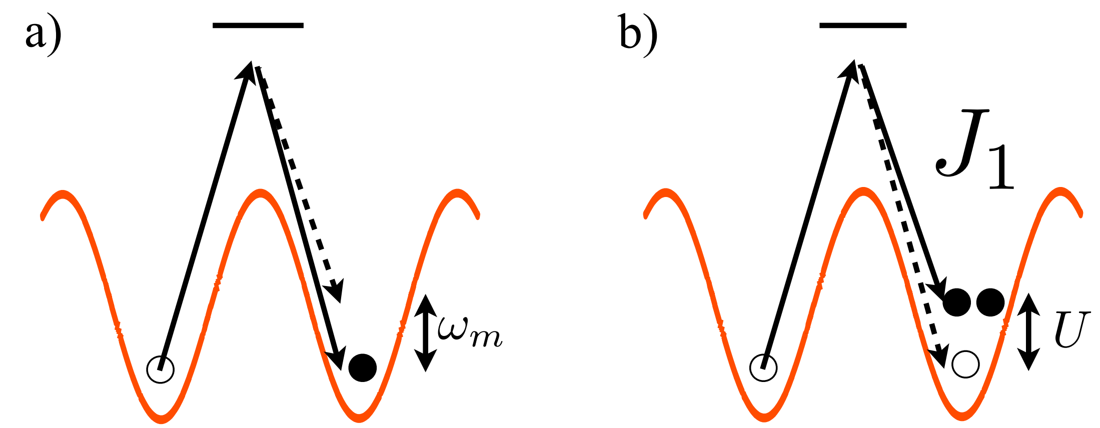
P. R. Johnson et al., New J. Phys. **11**, 093022 (2009)
S. Will et al., Nature **465**, 197 (2010

Modulation scheme:

- Raman dressing (non spin-changing) or lattice modulations (anharmonic)

$$U \gg J_0 \qquad\qquad J(t) = J_0 + J_1[\cos(\omega_{m1}t) + \cos(\omega_{m2}t)]$$





$$H_{\text{eff}} = \frac{U_{\text{eff}}^{(2)}}{2}\sum_i n_i(n_i-1) + \frac{U_{\text{eff}}^{(3)}}{6}\sum_i n_i(n_i-1)(n_i-2)$$
$$-J_0'\sum_{\langle i,j\rangle,n} b_i^\dagger b_j P_{i=n,j=n+1} - J_1\sum_{\langle i,j\rangle} b_i^\dagger b_j,$$

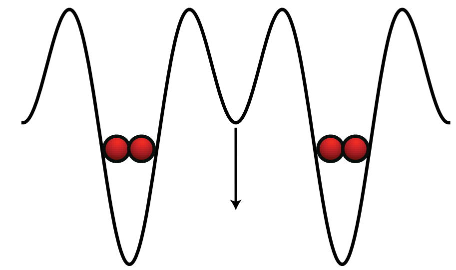
$$U_{\text{eff}}^{(2)} = -\Delta$$
$$U_{\text{eff}}^{(3)} = \delta U_3$$
$$J_0' = J_0 - J_1$$

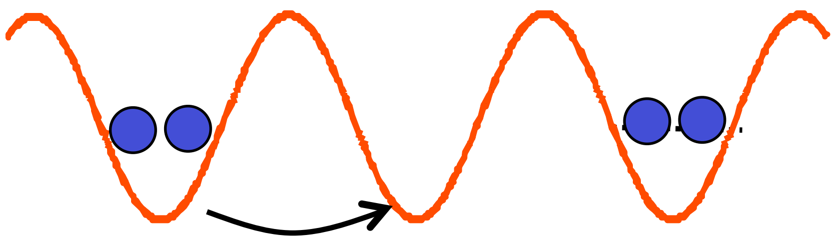
- Detuning controls two-body interactions
- Three-body interaction can become dominant

A. J. Daley and J. Simon, Phys. Rev. A 89, 053619 (2014)

Adiabatic state preparation for dimer states:

- Strongly interacting limit



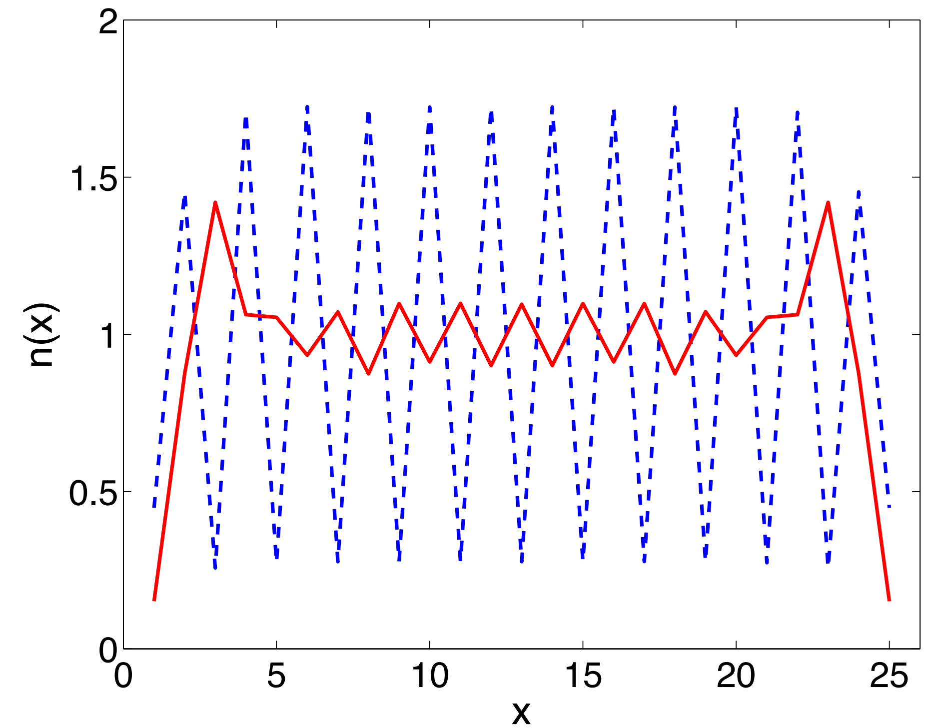


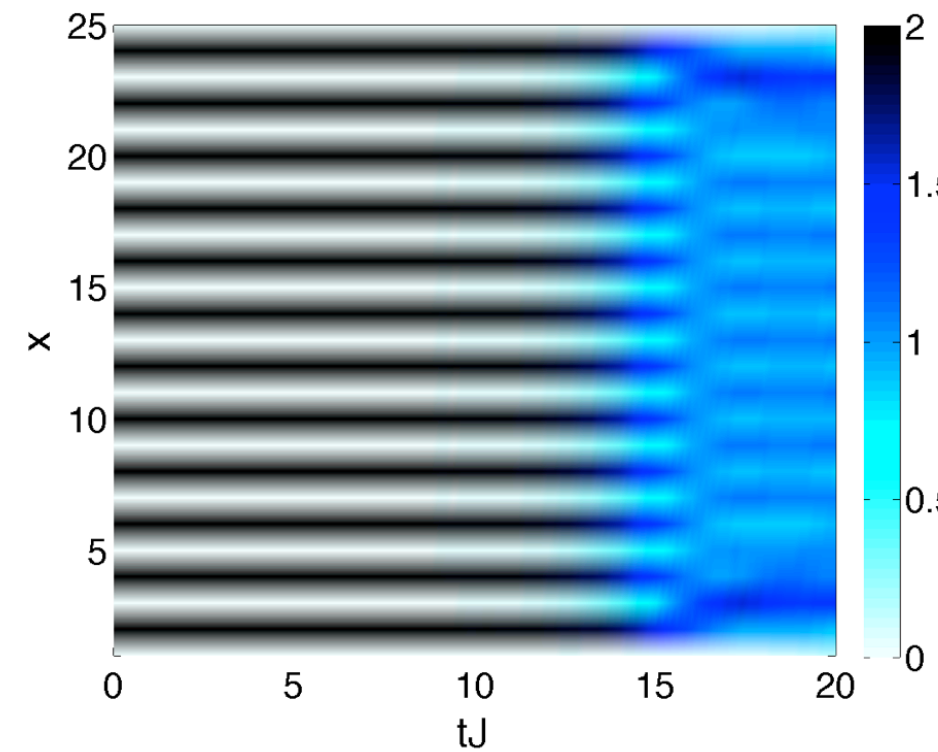
J_d $\quad d_l = (b_l)^2$

$$H_d = \sum_{\langle i,j\rangle} \left(-J_d d_i^\dagger d_j + U_d d_i^\dagger d_i d_j^\dagger d_j + 2\varepsilon_j d_j^\dagger d_j \right)$$

$$J_d = \frac{2J^2}{U_{\rm eff}^{(2)}}, \; U_d = \frac{4J^2}{U_{\rm eff}^{(2)}} - \frac{4J^2}{U_{\rm eff}^{(3)}}$$

Control over dimer-dimer interactions



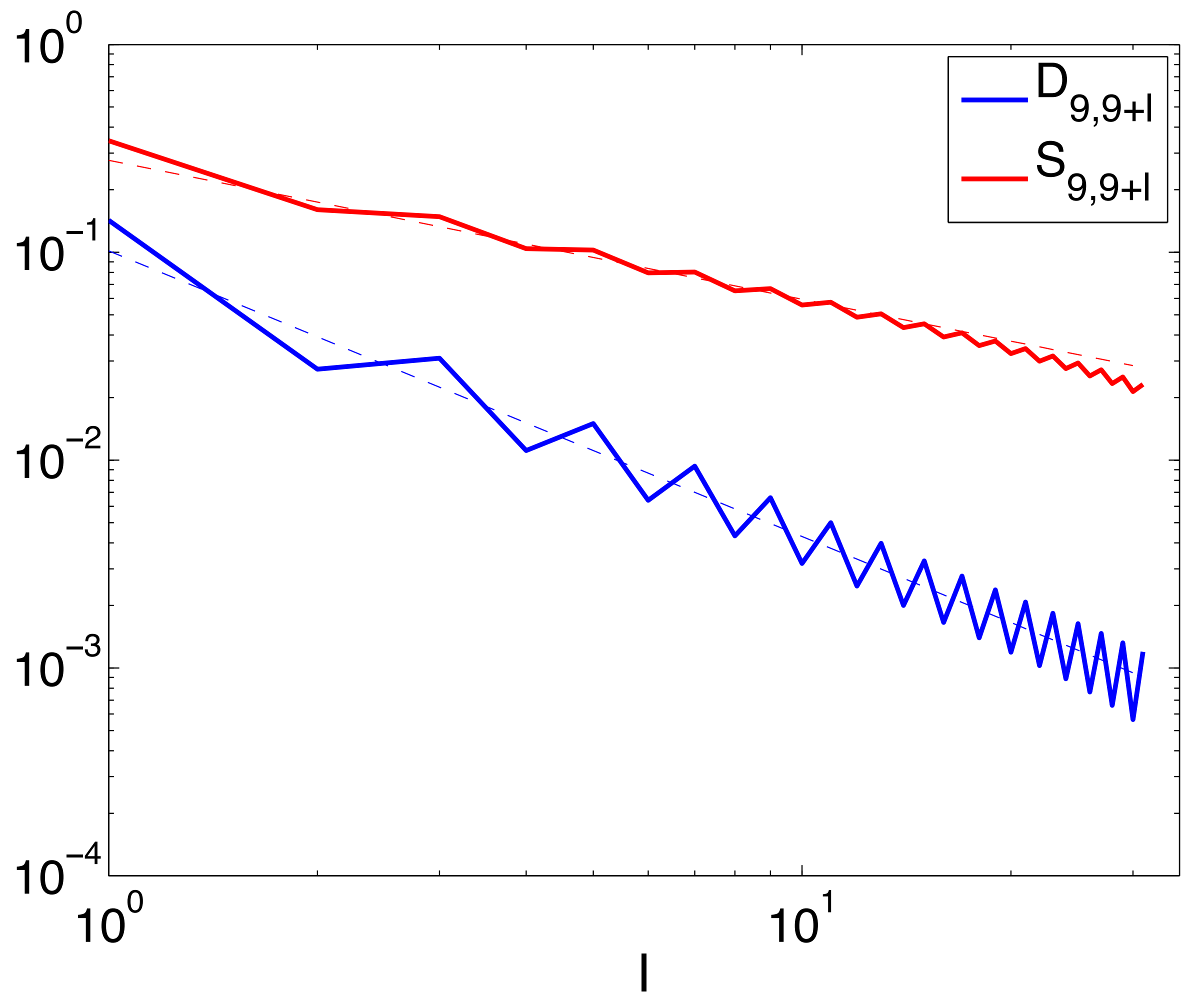


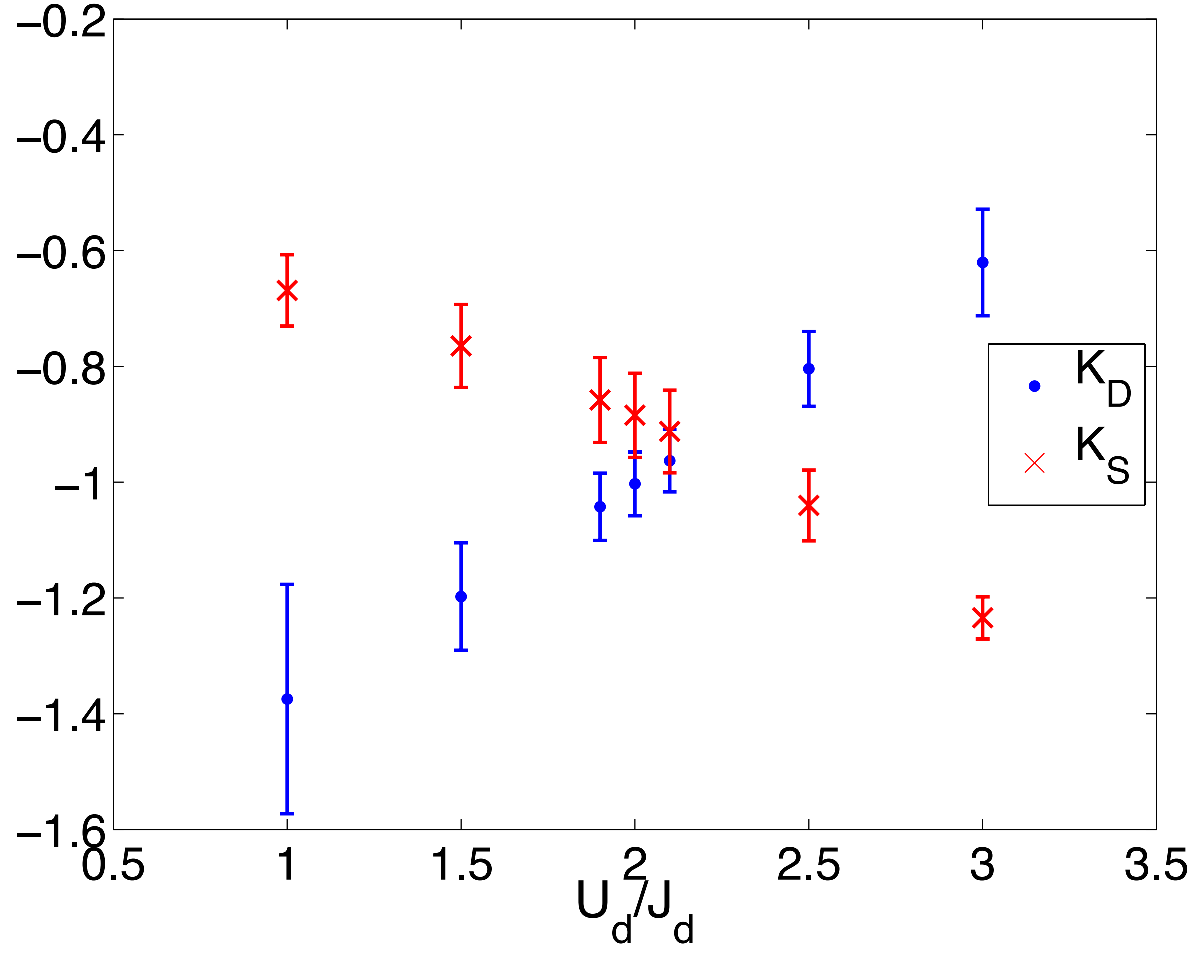
A. J. Daley and J. Simon, Phys. Rev. A 89, 053619 (2014)

Dimers at half filling in 1D: Crossover charge-density wave vs. superfluid

$$H_d = \sum_{\langle i,j\rangle} \left(-J_d d_i^\dagger d_j + U_d d_i^\dagger d_i d_j^\dagger d_j + 2\varepsilon_j d_j^\dagger d_j \right) \qquad J_d = \frac{2J^2}{U_{\rm eff}^{(2)}},\ U_d = \frac{4J^2}{U_{\rm eff}^{(2)}} - \frac{4J^2}{U_{\rm eff}^{(3)}}$$

$$S_{i,j} = \langle d_i^\dagger d_j \rangle \qquad D_{i,j} = \langle d_i^\dagger d_i d_j^\dagger d_j \rangle - \langle d_i^\dagger d_i \rangle \langle d_j^\dagger d_j \rangle$$



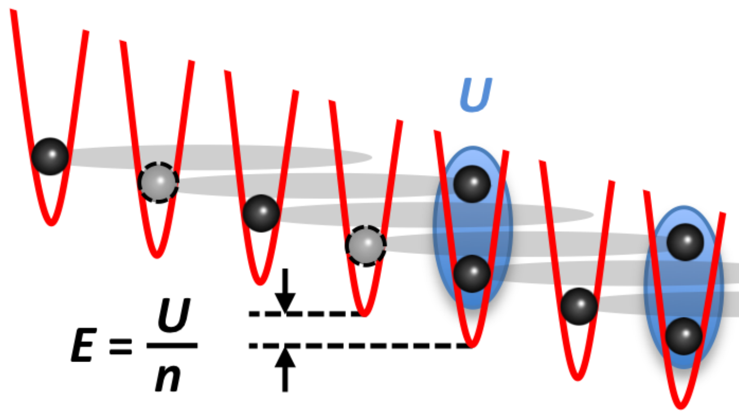


S. Diehl, M. Baranov, A. J. Daley, and P. Zoller, Phys. Rev. Lett. **104**, 165301 (2010)
A. J. Daley and J. Simon, Phys. Rev. A 89, 053619 (2014)

Related ongoing work

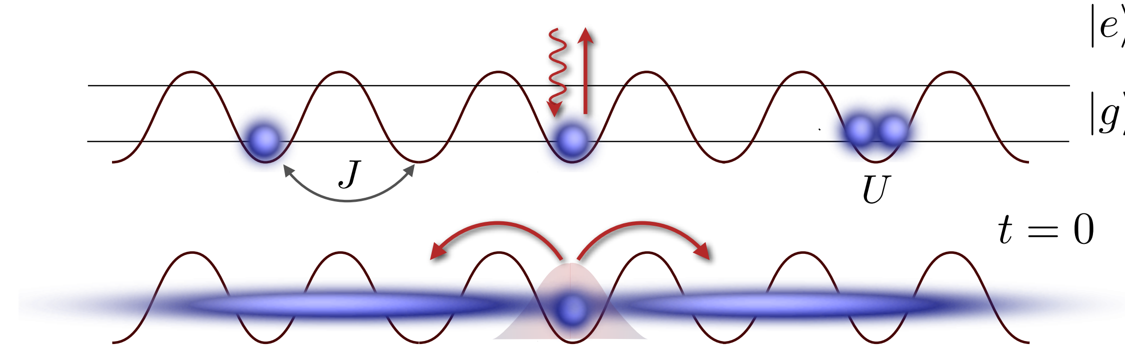
Ongoing work at Strathclyde: Theory of coherent / dissipative dynamics

Transport dynamics in optical lattices



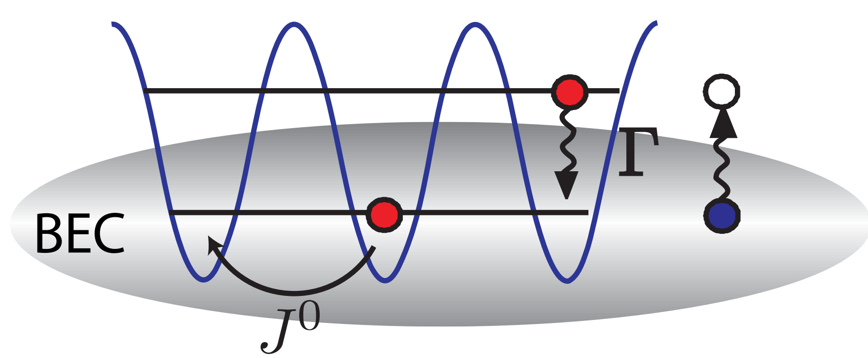
F. Meinert et al., PRL **111**, 053003 (2013)
F. Meinert et al., Science 344, 1259 (2014)

Dissipation and Thermalization



J. Schachenmayer, L. Pollet, M. Troyer, A. J. Daley,
Phys. Rev. A **89**, 011601(R) (2014)

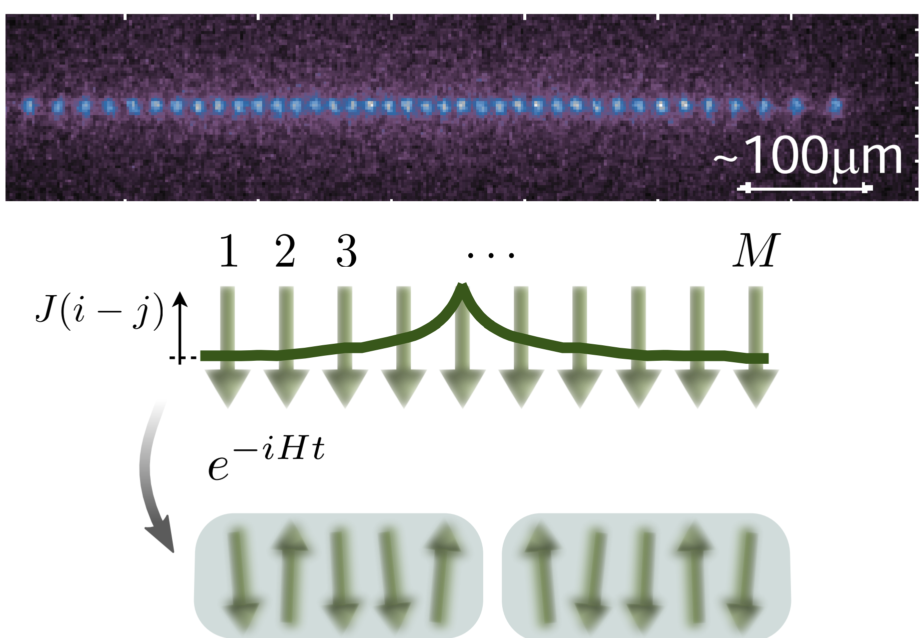
Cooling / dissipative state preparation



Dark State Cooling:
A. Griessner, A. J. Daley, S. R. Clark, D. Jaksch, P. Zoller,
PRL **97**, 220403 (2006)
Dissipative d-wave pairing:
S. Diehl, W. Yi, AJD and P. Zoller
PRL **105**, 227001 (2010)

Now: Extensions to more complex reservoirs,
non-markovian dynamics

Quenches and Long-range interactions



J. Schachenmayer et al., PRX 3, 031015 (2013)

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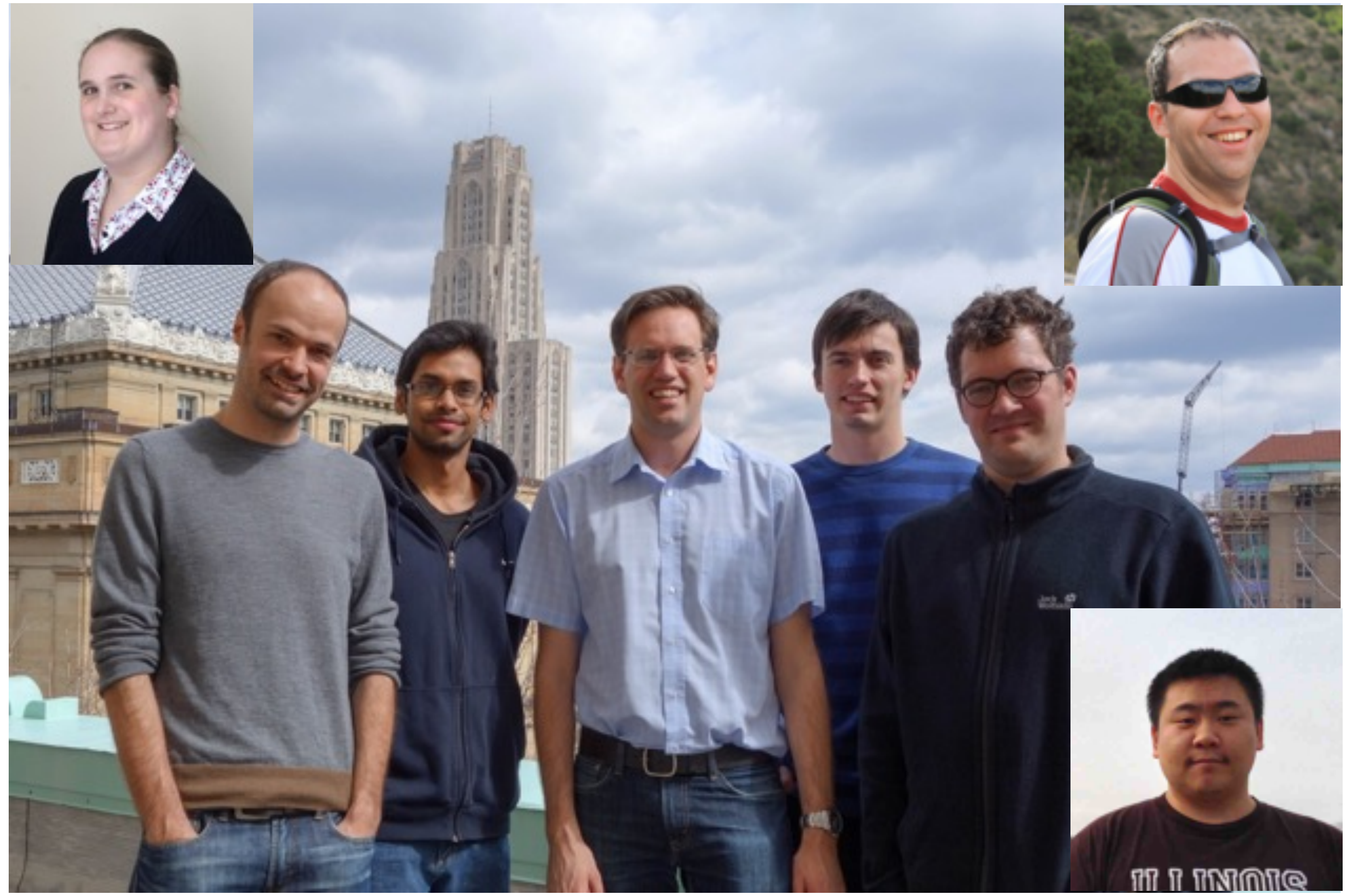

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AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
UNITED STATES AIR FORCE

University of Strathclyde Glasgow

Summary / Outlook

- Atoms in optical lattices offer new tools to prepare and observe many-body states
- Long-lived Metastable states can be controllably prepared with time-dependent dynamics
- Dissipative dynamics opens new directions, e.g., via quantum Zeno effect, and dissipative driving