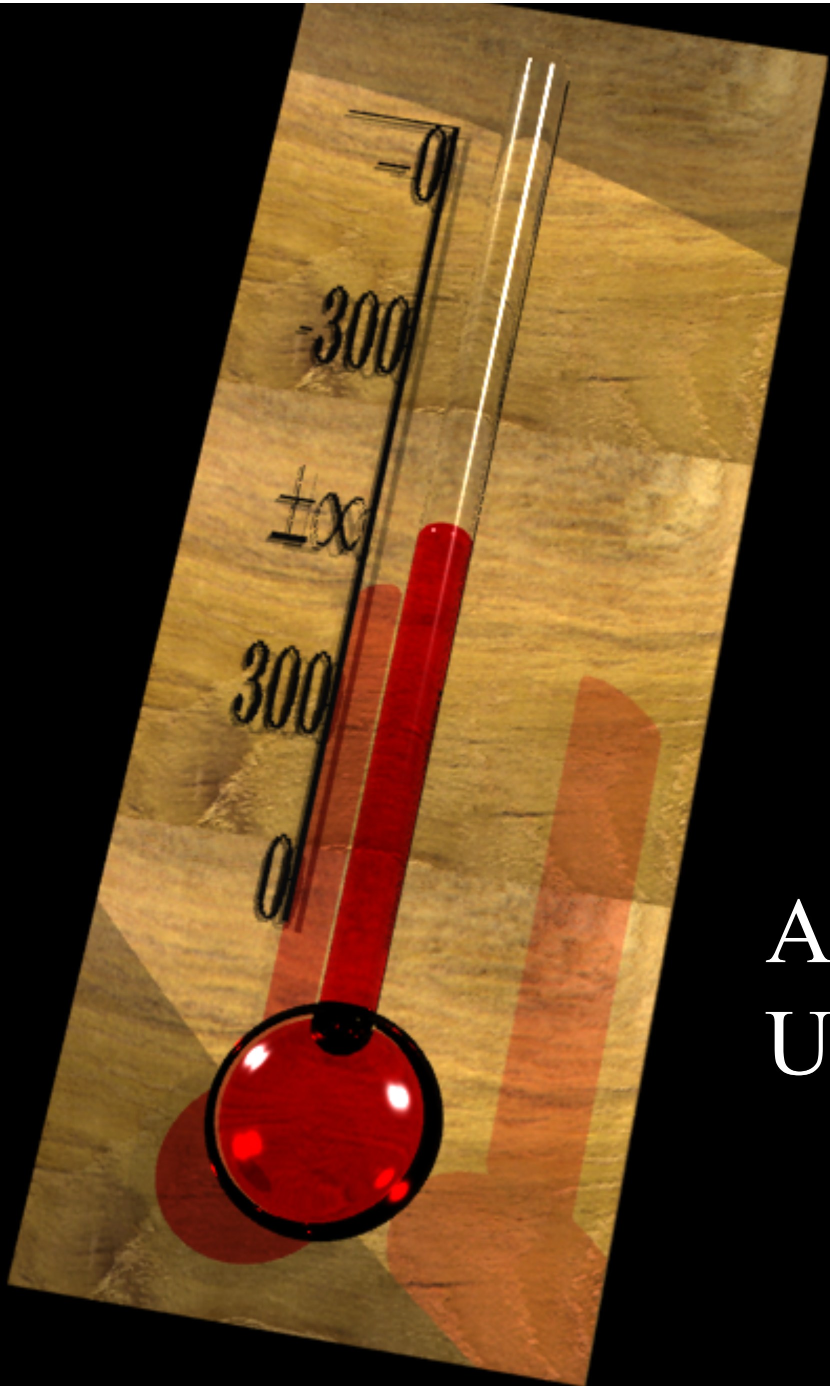
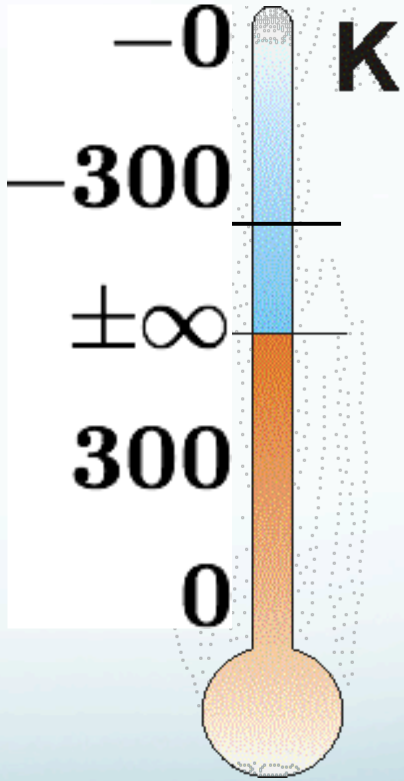


Allard Mosk
University of Twente

Atomic gases at negative Temperature



Allard Mosk

Complex Photonic Systems

MESA+ Institute for Nanotechnology

University of Twente, The Netherlands

Glasgow, 23 Oct 2014

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Outline

What is negative temperature ?

Stability criteria

Production

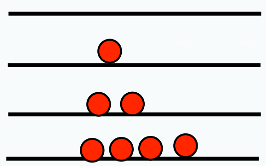
How real is negative temperature?

(Which entropy function to use)

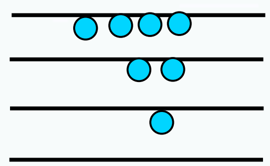
What doesT<0 mean ?

Boltzmann distribution:

$$n(E) = n_0 e^{-E/kT}$$



T>0



T<0

"T<0" only makes sense if *E* has an upper limit.

What does T mean anyway?

First Law of Thermodynamics:

$$dQ = TdS + \cancel{dW}$$

Choose conditions so that $dW = 0$

$$T = \frac{1}{(dS/dQ)_W}$$

Thermodynamic definition of temperature

Why is $T<0$ interesting

Einstein (1905) stated the fluctuation-dissipation theorem:

cannot be negative

Variance of the thermal noise

is proportional to

can be negative

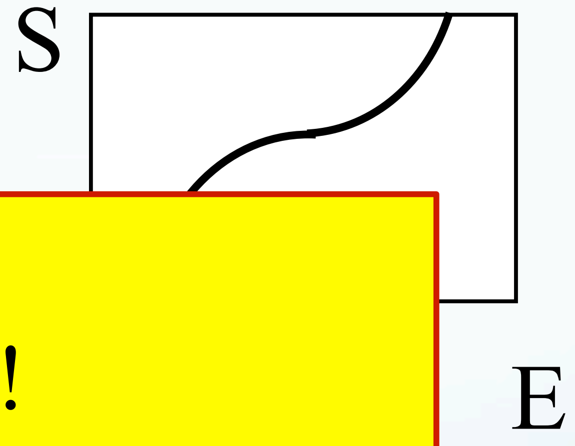
kT x The damping time constant

must have same sign as T

A similar (more general) theorem w[...] developed by Nyquist to describe Johnson noise

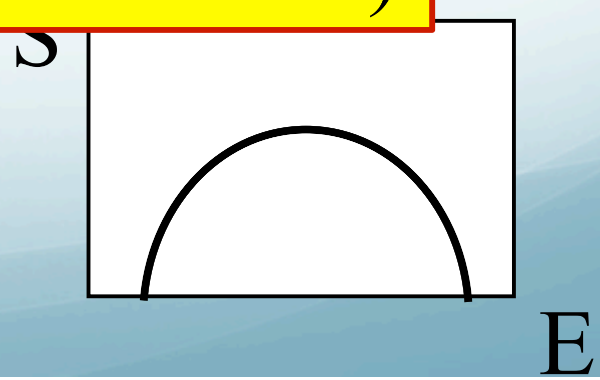
Entropy vs. Energy

An atom has unbounded kinetic energy.



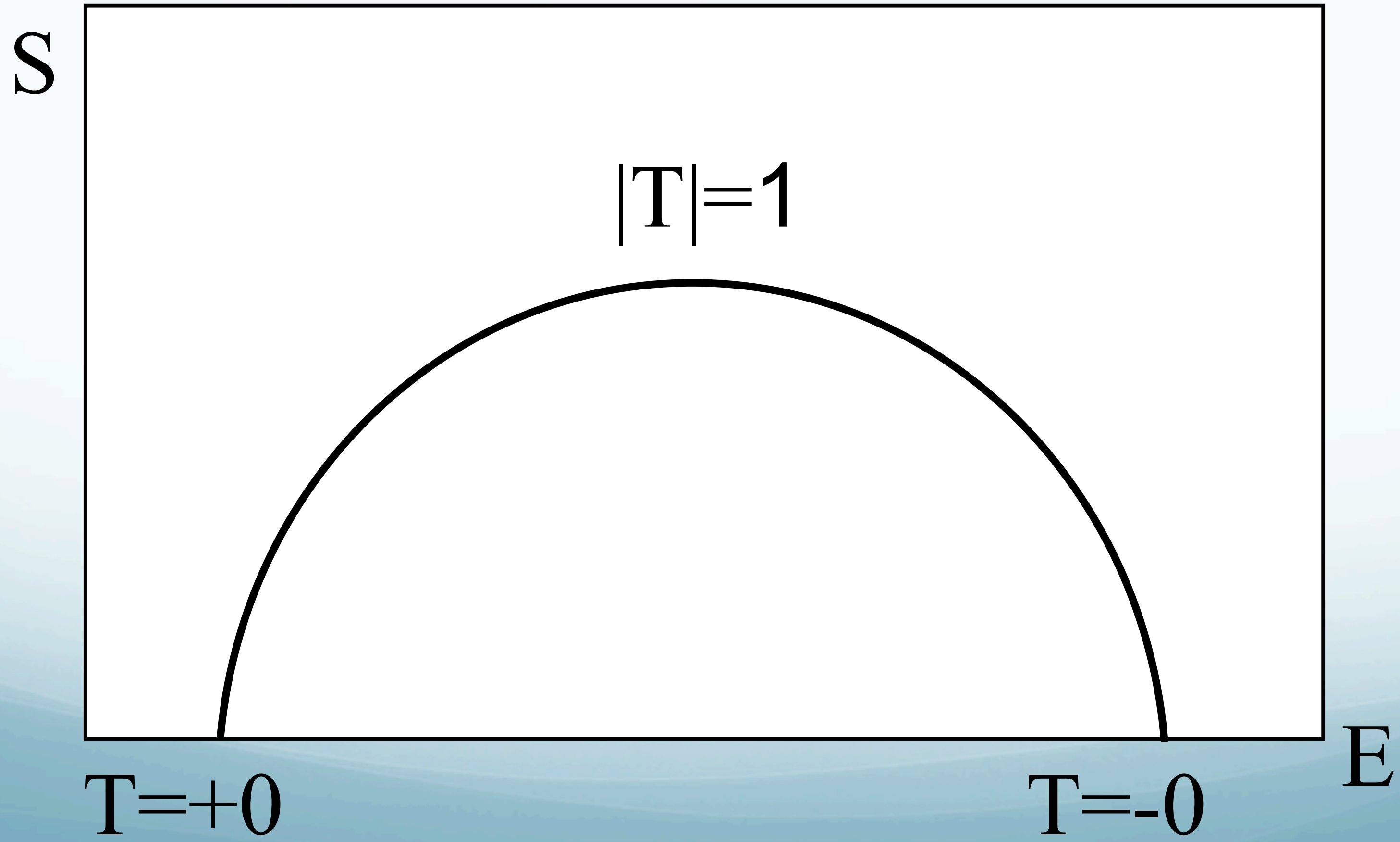
Wha

Does the entropy go down?
Yes, the entropy goes down!
(Frenkel & Warren, ArXiv 1403.4299)



Bounded energy: S becomes nonmonotonic function of E

$$T = \frac{1}{(dS/dQ)_W}$$



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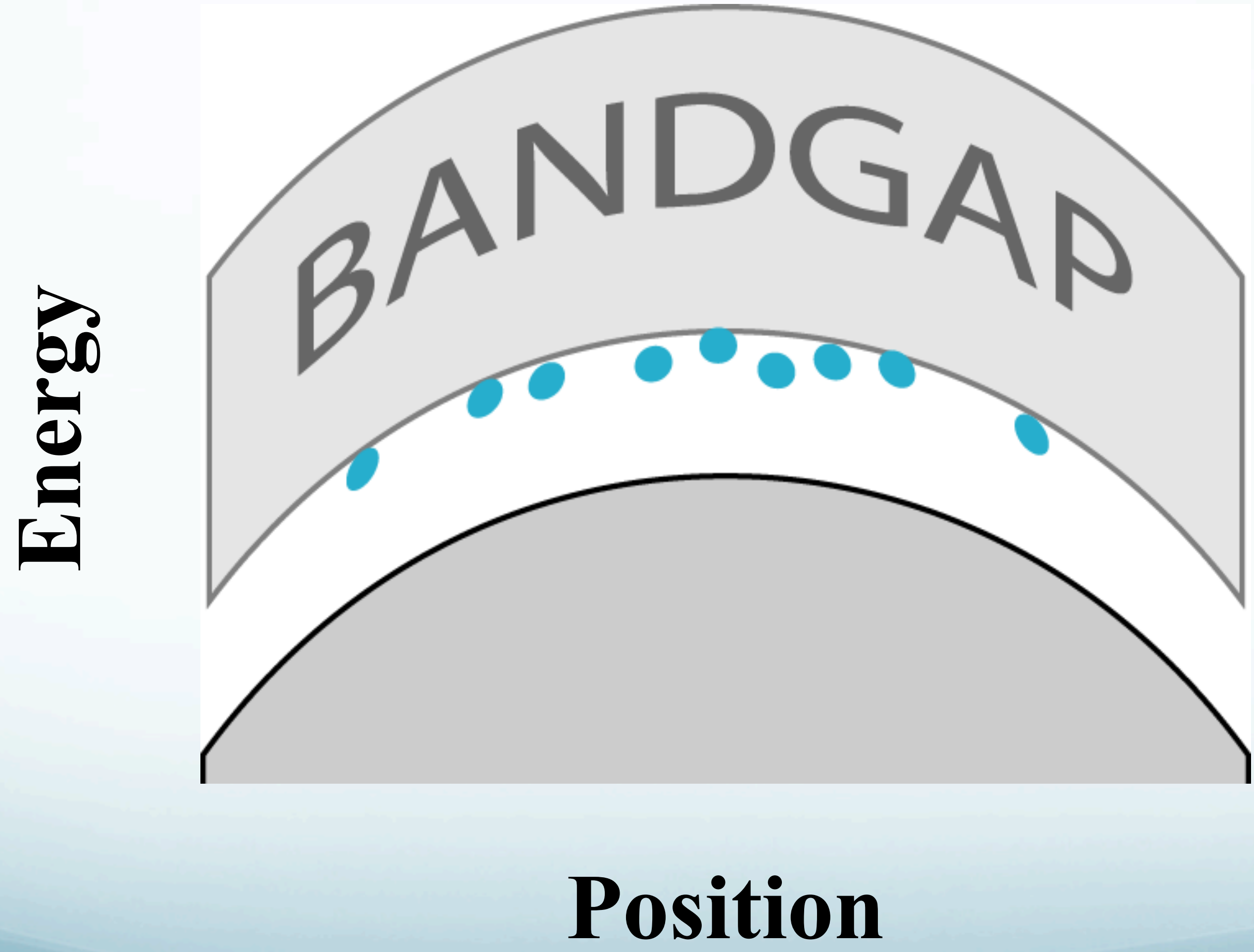
Bandgap enforces Maximum kinetic energy



Which system to choose

Electrons in semiconductors ?

No, too much coupling to lattice vibrations.
Not possible to measure distribtion directly

Photons in photonic crystals

No, interactions too weak.
Interesting new options include waveguide arrays, PC fibers

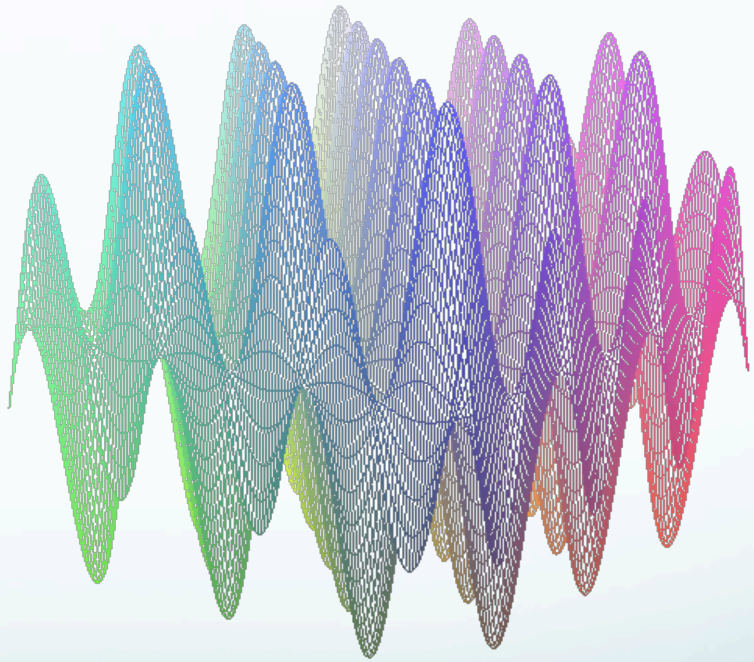
Atoms in optical lattices

Is it possible to have a 3D bandgap ?

Can we prevent interband scattering ?

3D optical lattices

The bandgap
can be provided by
an optical lattice



$$V(x, y, z) = V_0 \left[\cos(kx) + \cos(ky) + \cos(kz)\right]$$

Schrödinger Equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V_0\cos^2(k_0x)\right]\Psi(x)=E\Psi(x)$$

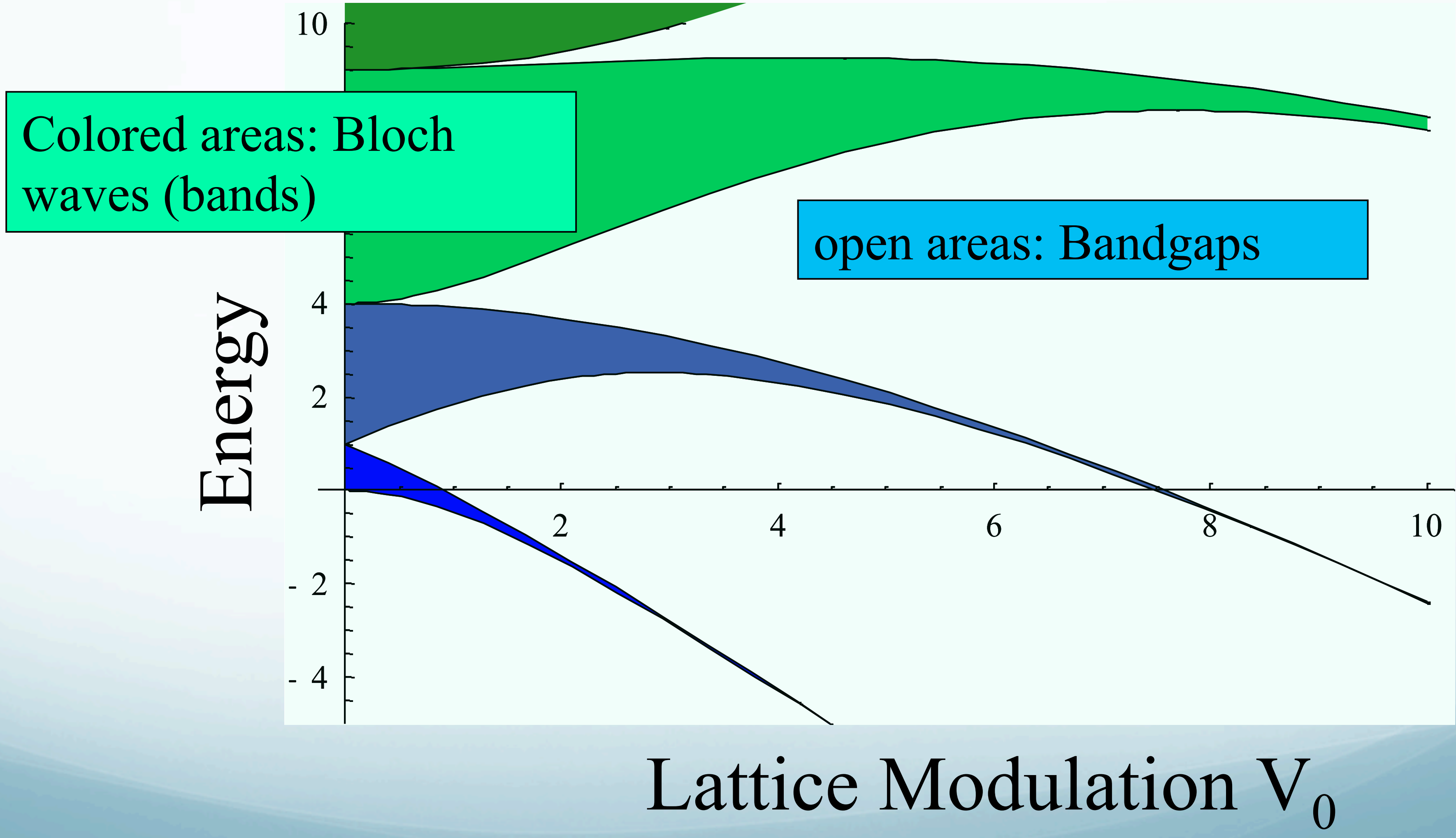
With suitable transformations:

$$\boxed{\frac{m}{\hbar^2k_0^2}V_0 \longrightarrow q} \qquad \boxed{k_ox/2 \longrightarrow z} \qquad \boxed{\frac{m(E-V_0/2)}{\hbar^2k_0^2} \longrightarrow a}$$

$$\left[\frac{d^2}{dz^2}+a-2q\cos(2z)\right]\Psi(z)=0$$

MATHIEU Emile Léonard, 1835-1890

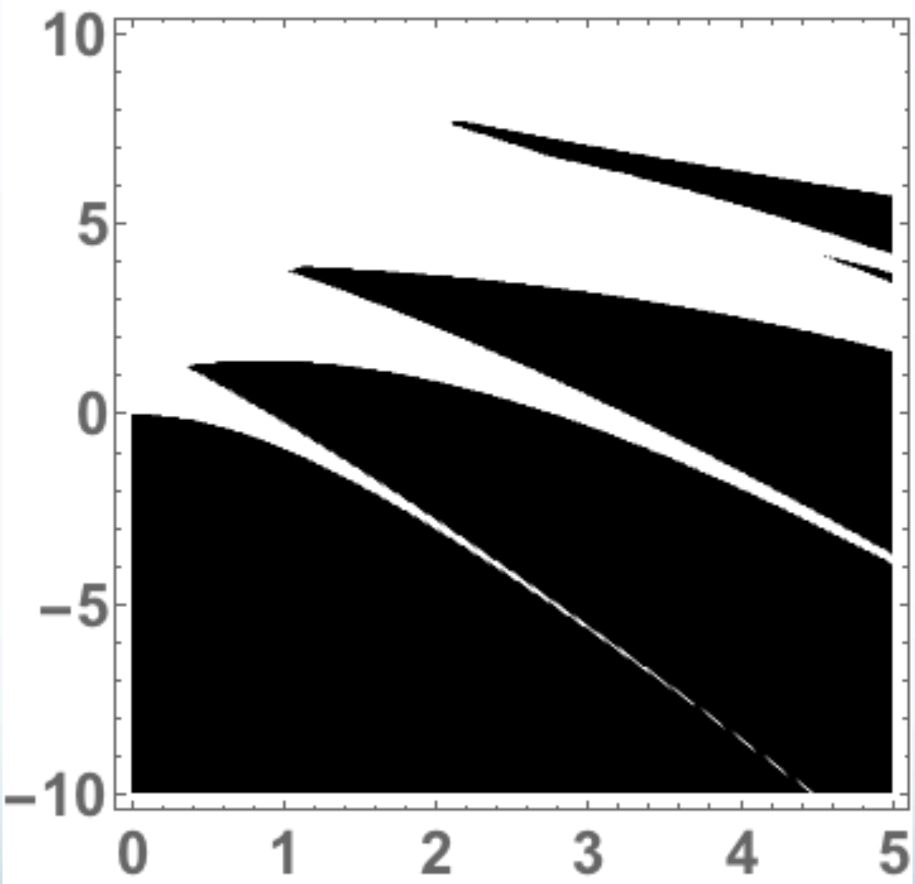
Bands of a 1D optical lattice



Bandgaps for atoms

In 1D the Mathieu equation has a bandgap for any nonzero lattice potential.
In 2D the x,y bands overlap and a minimum lattice strength is needed.

2D Band structure of the Mathieu Equation

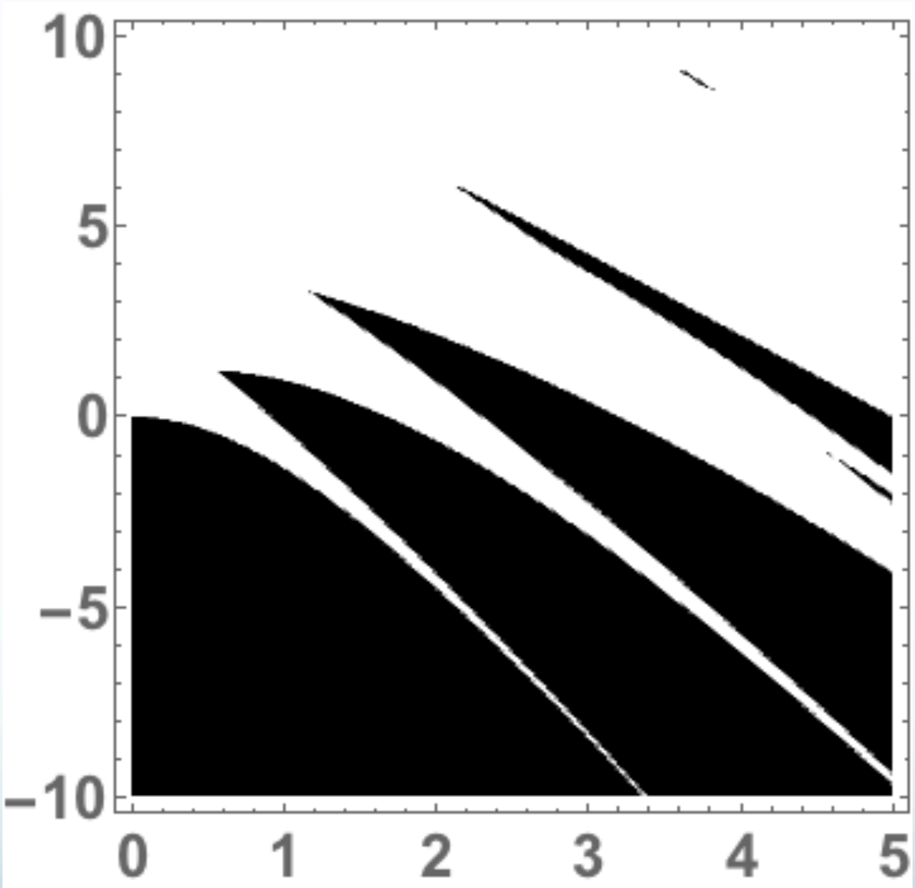


Energy a

Lattice potential q

3D Band structure of the Mathieu Equation

Energy a



Lattice potential q

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How to get to top of band

First proposals to produce a condensate at the top of the band (with negative mass states) were based on

- Phase imprinting (Pu *et al.*, PRA **67**, 043605, 2003)
- Acceleration pulse (Zheng *et al.*, PRL **93**, 230401, 2004)

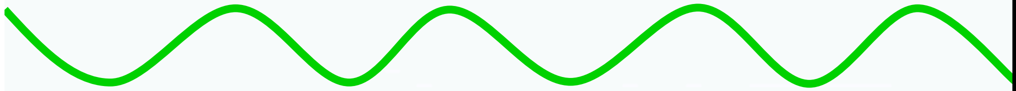
These methods require an accurately calibrated acceleration/ phase pulse and are therefore very sensitive to noise, timing and calibration error. Moreover, they lead to considerable entropy production.

Mott phase transition

Weak lattice potential:

Eigenstate   Bloch waves

Interactions are a perturbation



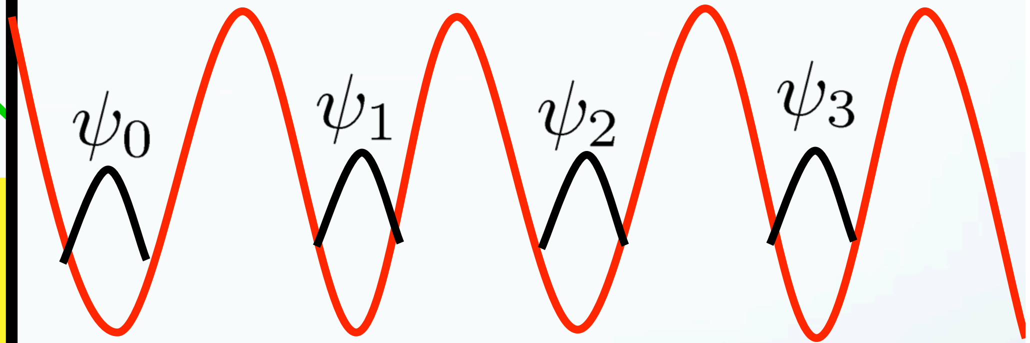
$$\Psi_0(\mathbf{x}_1, \mathbf{x}_2, ...\mathbf{x}_N) \approx$$

$$\psi_{k=0}(\mathbf{x}_1)\psi_{k=0}(\mathbf{x}_2)...\psi_{k=0}(\mathbf{x}_N)$$

Strong lattice potential:

Eigenstate   Wannier states

Hopping is a perturbation

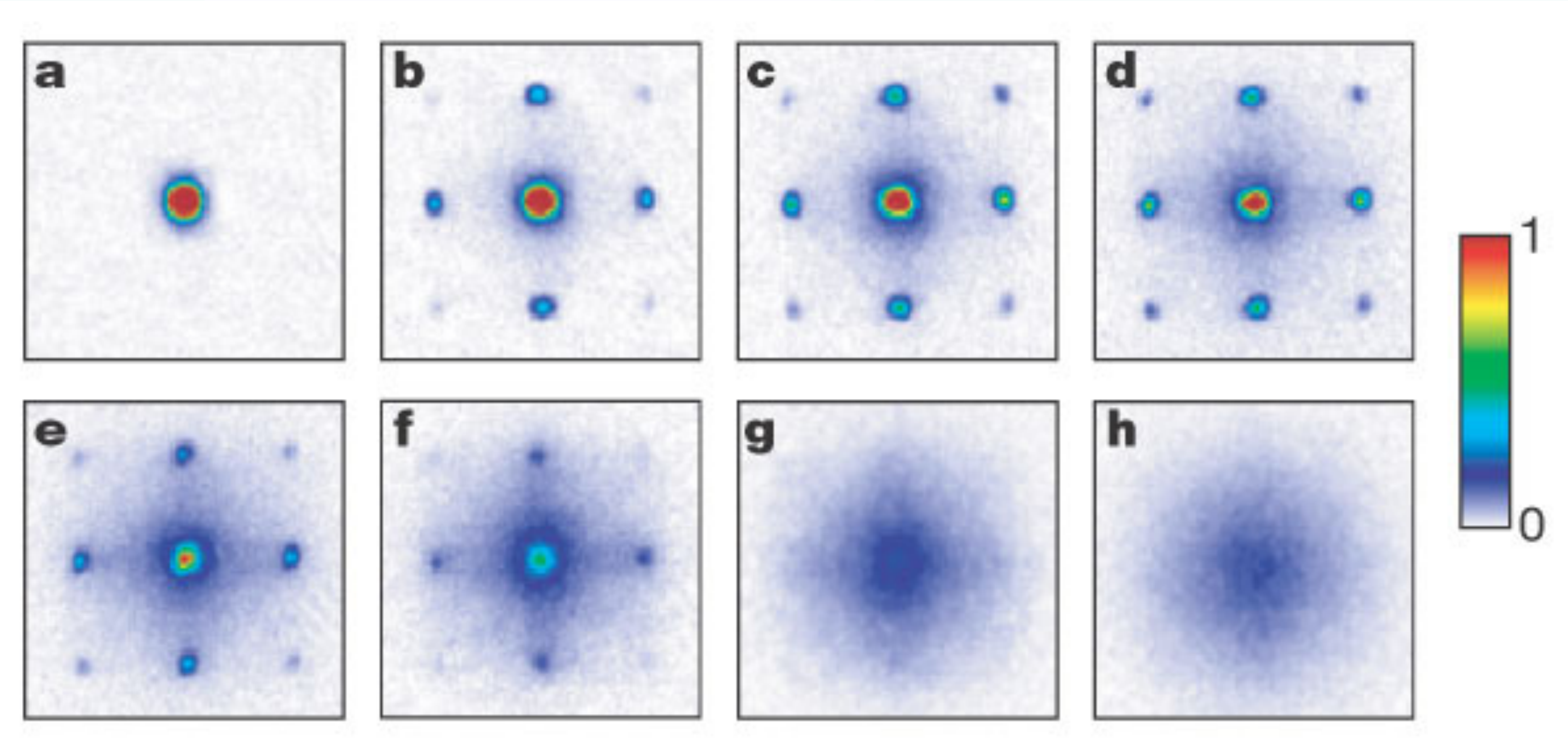


$$\Psi_0(\mathbf{x}_1, \mathbf{x}_2, ...\mathbf{x}_N) \approx$$

$$\sum_{\text{sym}} \psi_0(\mathbf{x}_1)\psi_1(\mathbf{x}_2)...\psi_N(\mathbf{x}_N)$$

Mott phase transition

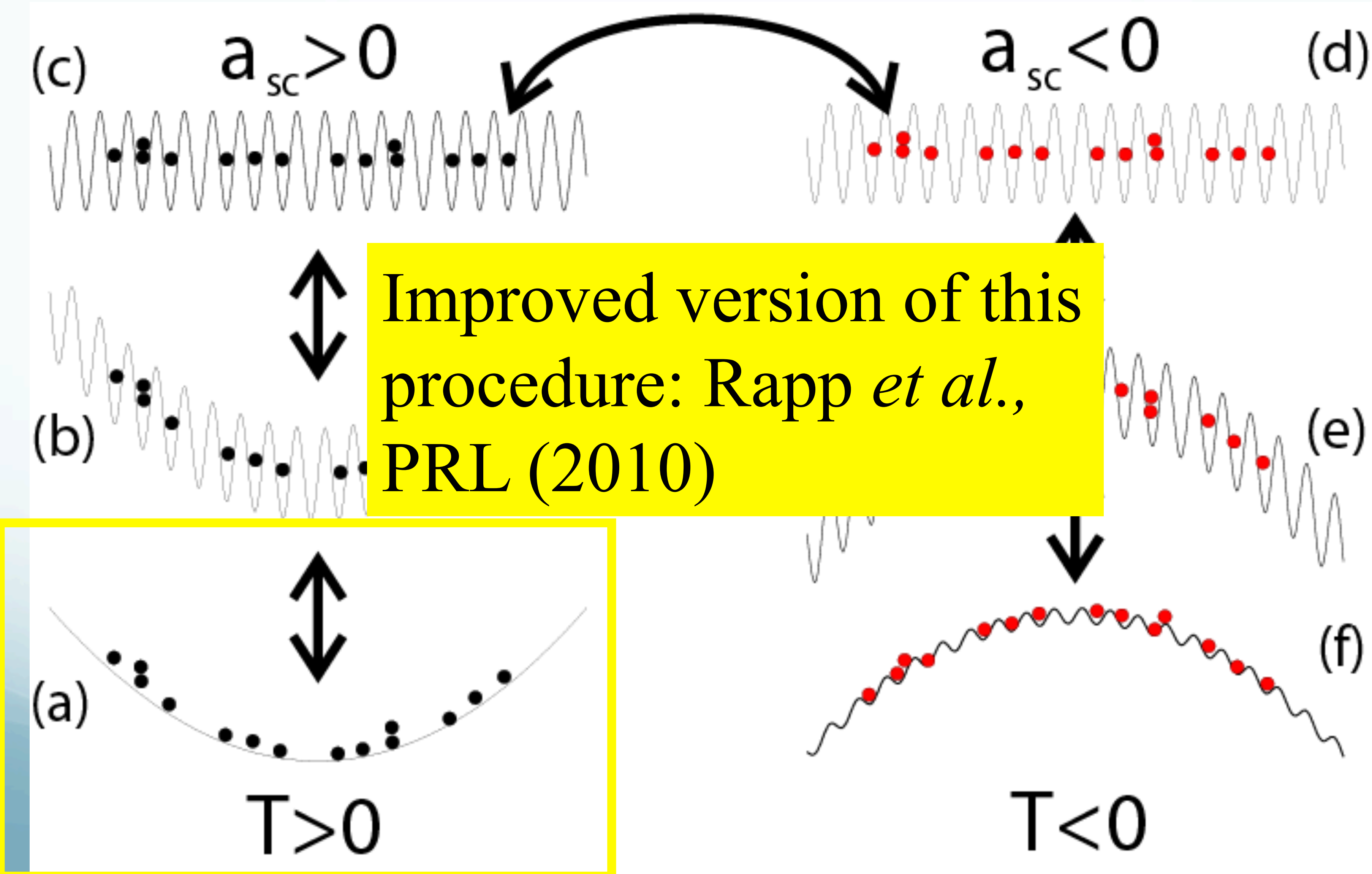
- (M. Greiner et al.,Nature **415**,39, 2002)



Production of negative T

- Make a cold gas (BEC) at T>0 with repulsive interactions
- Apply a lattice, make the Mott transition
  - The Mott state is now the lowest many-particle state in the first band
- Change the interactions to attractive
  - The Mott state is now the highest state in the first band
- Soften the Mott insulator by decreasing the lattice slowly
  - Adiabatic: We stay in the highest state in the band!

Production of negative T



Improved version of this procedure: Rapp *et al.*, PRL (2010)

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How real is negative temperature?

- NT can be in equilibrium with itself
- NT can be in equilibrium with other NT
- NT restores equilibrium distribution when perturbed
- NT Cannot be in equilibrium with macroscopic object
- NT is metastable on a time scale of seconds/minutes

How real is temperature of a sub- μK trapped gas?

- TG can be in equilibrium with itself
- TG can be in equilibrium with other TG
- TG restores equilibrium distribution when perturbed
- TG Cannot be in equilibrium with macroscopic object
- TG is metastable on a time scale of seconds/minutes

Conclusion

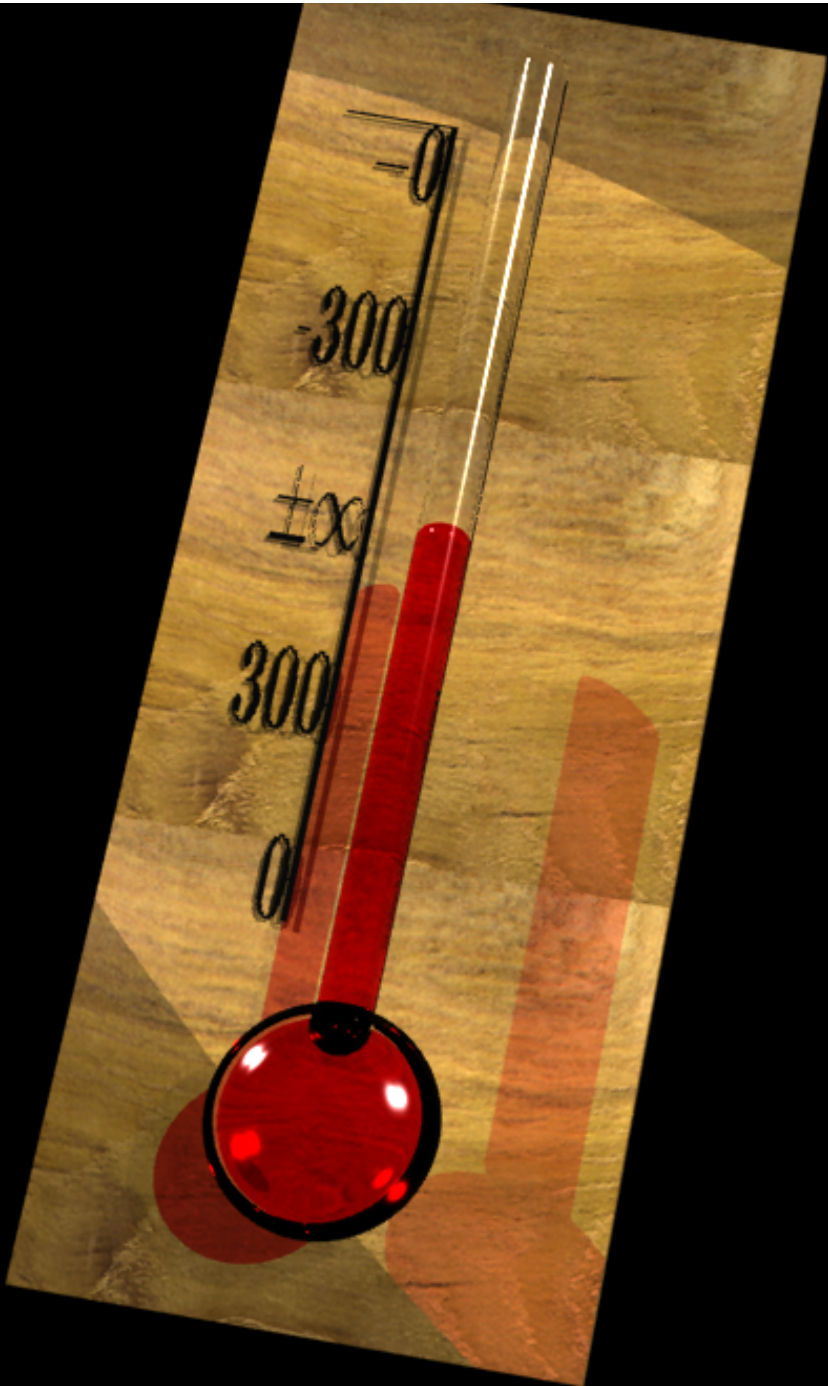
A Trapped Negative temperature gas

- Is a new type of equilibrium matter.
- Can be created under realistic conditions.
- Is trapped by a potential maximum.
- Has negative viscosity.

First proposal: APM, PRL **95**, 040403 (2005)

More quantitative theory: Rapp et al., PRL **105**, 220405 (2010)

Experiment: Braun et al., Science **339**, 52 (2013)



Allard Mosk
University of Twen

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Which entropy function to use?

- Boltzmann entropy $$S = k \log W$$

Confusion may arise on the definition of W especially for small systems

- Gibbs entropy $$S = -k \sum_i p_i \log p_i$$

This is the form of the entropy function that is supported by information theory (Shannon, Jaynes, Landauer, Reza, MacKay).

If the number of particles is large enough, there is no appreciable difference between the definitions.

Power-law density of states

- Gibbs entropy $$S = -k \sum_i p_i \log p_i$$

can be integrated if the density of states follows a power law and the distribution is thermal, to yield

$$S_{\mathrm{powerlaw}} = k \log \Omega(E) + \mathrm{constant}$$

Free and trapped particles and very many other systems have an effective power-law density of states.

It is a **mistake** to use the powerlaw entropy for non-powerlaw systems (Frenkel & Warren, Arxiv, 2014).