Arrays of waveguides with losses and cavity BEC

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Update. Atomic losses in a BEC in an Optical Lattice

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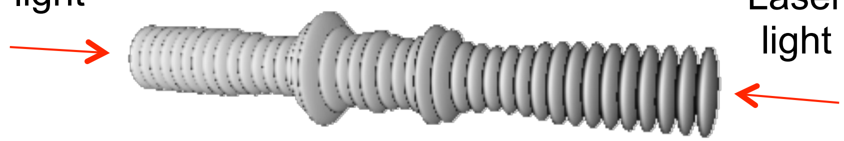
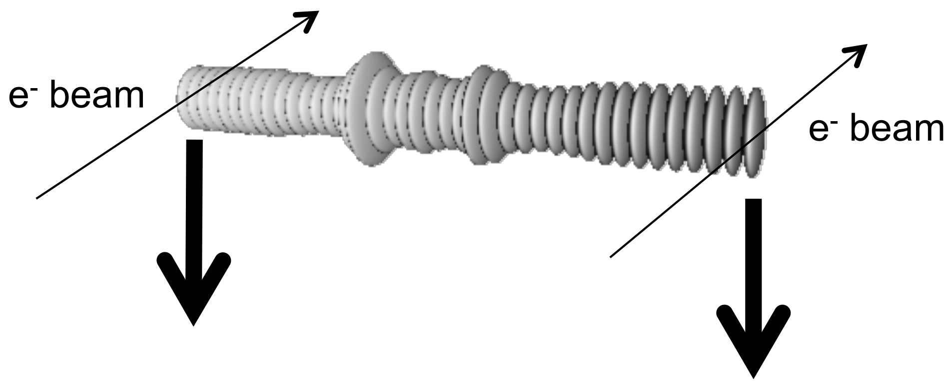
Conservative Case	Dissipative Case

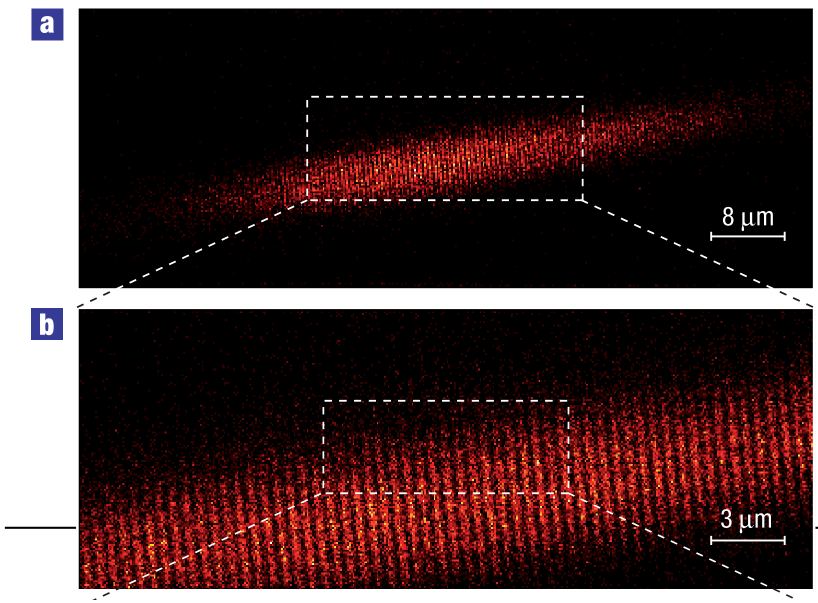
Confined BEC in an Optical Lattice

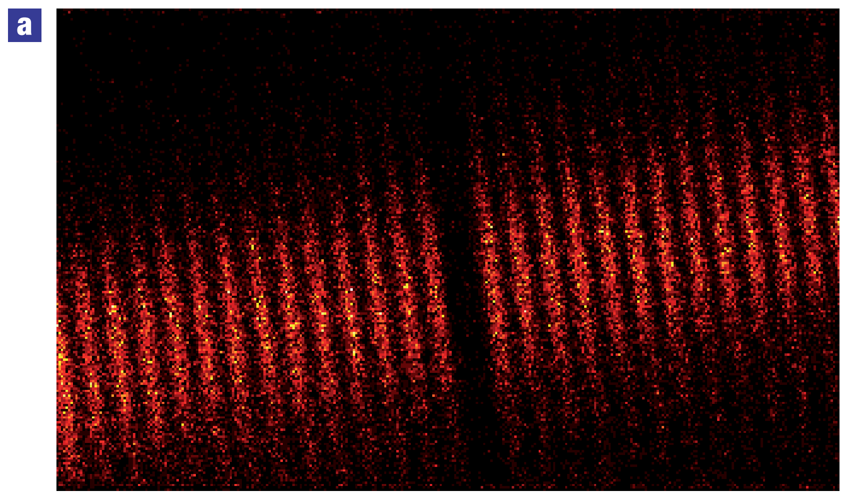


Gericke et al. Nat. Phys. **4**, 949 (2008)



Loss of Atomic Density at the Boundaries





University of Mainz

Arrays of optical waveguides

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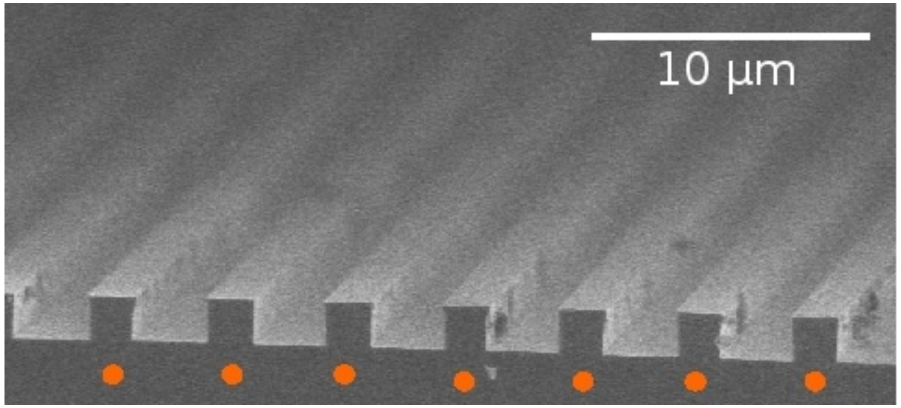
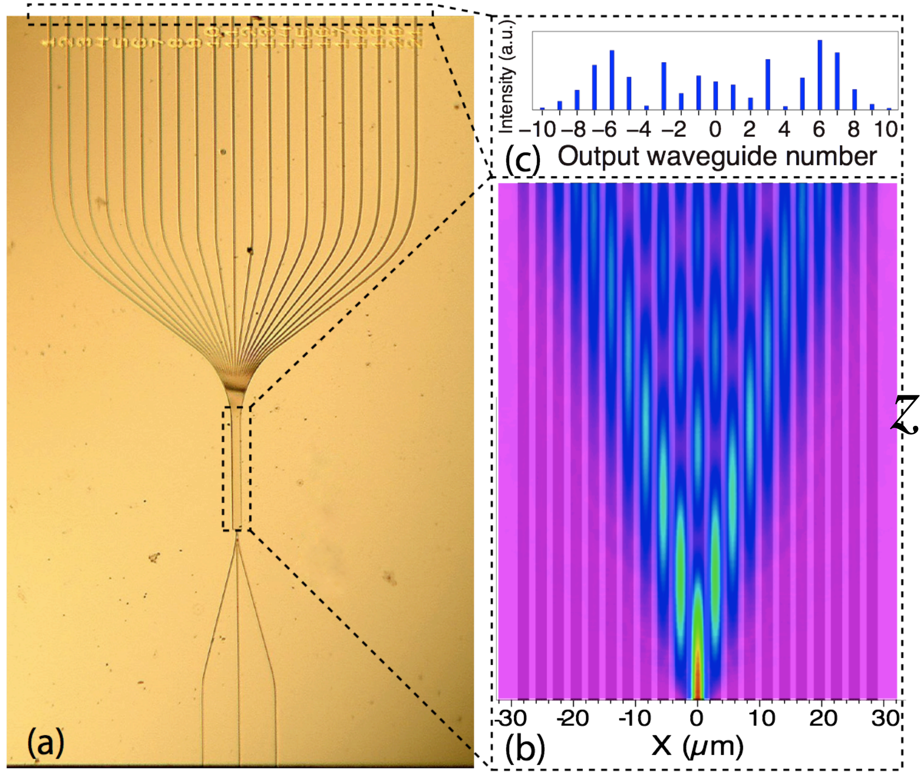


Arrays of optical waveguides with evanescent coupling are also well described by the Discrete Non Linear Shrödinger Equation (DNLSE)

$$i\frac{d}{dz}\psi_m + (\psi_{m+1} + \psi_{m-1}) + \nu |\psi_m|^2 \psi_m = 0$$

Nearest Neighbour Coupling

Self-Focusing Self-Defocusing



A Peruzzo et al., Science **329**, 1500 (2010)

Quantum Walks of Correlated Photons

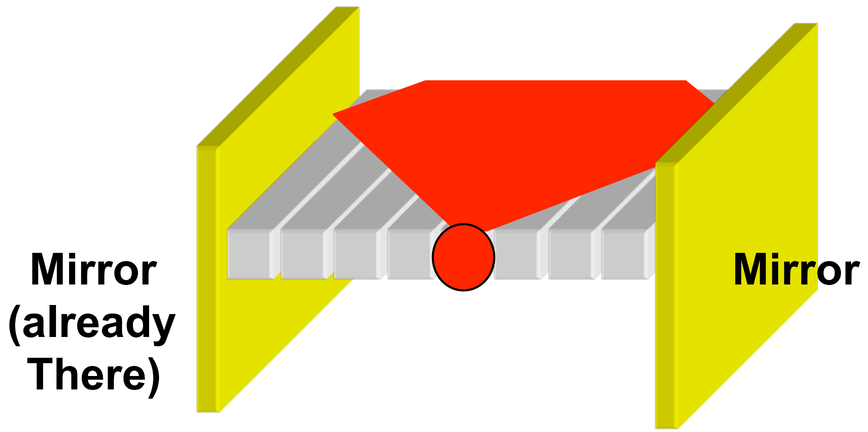
Large Scale Quantum Interference

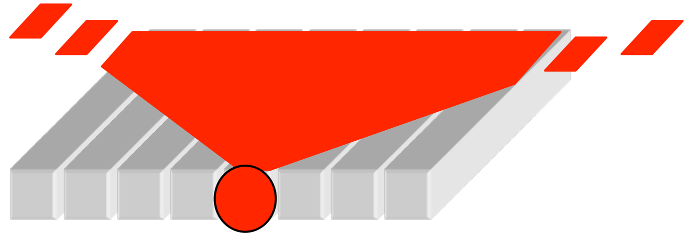
Quantum Correlations violating classical limits by 76 standard deviations.

SiON

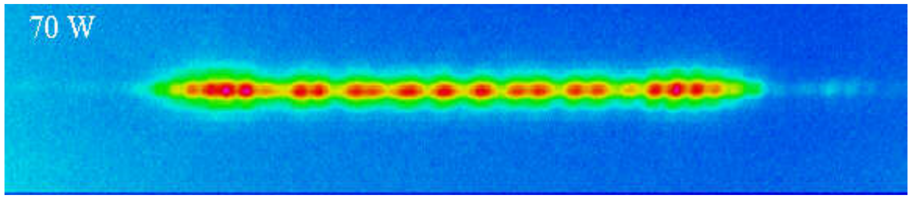
Conservative Case | **Dissipative Case**

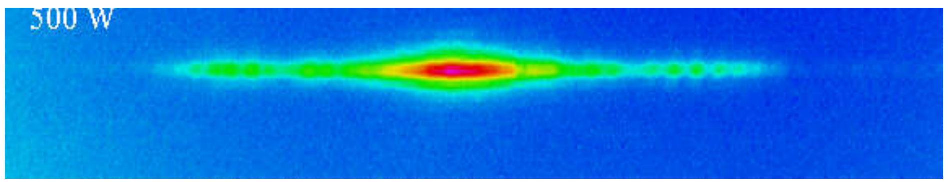
Propagation in a waveguide array





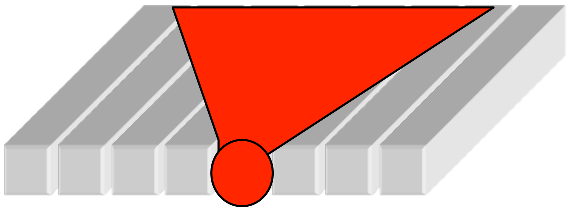
Loss of Light at the Boundaries



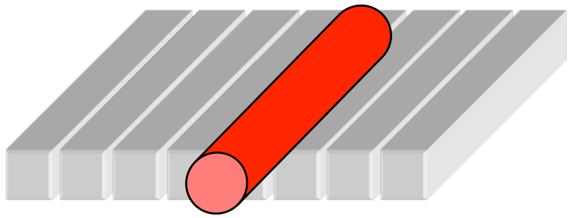


m

m



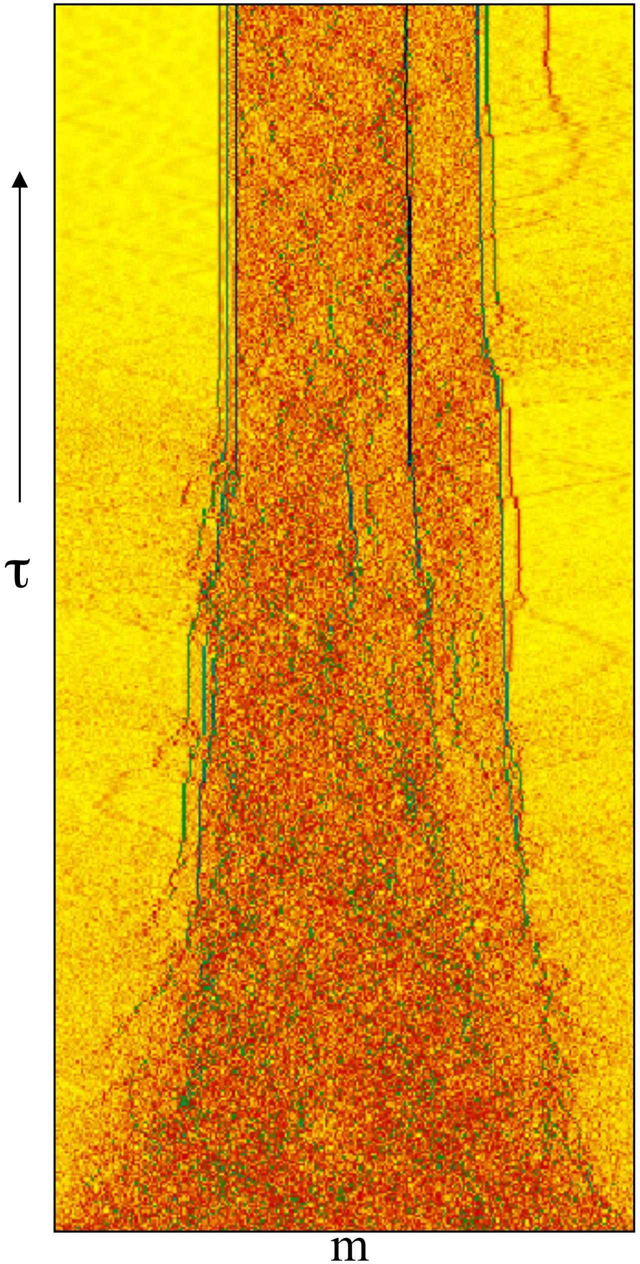
Low input power

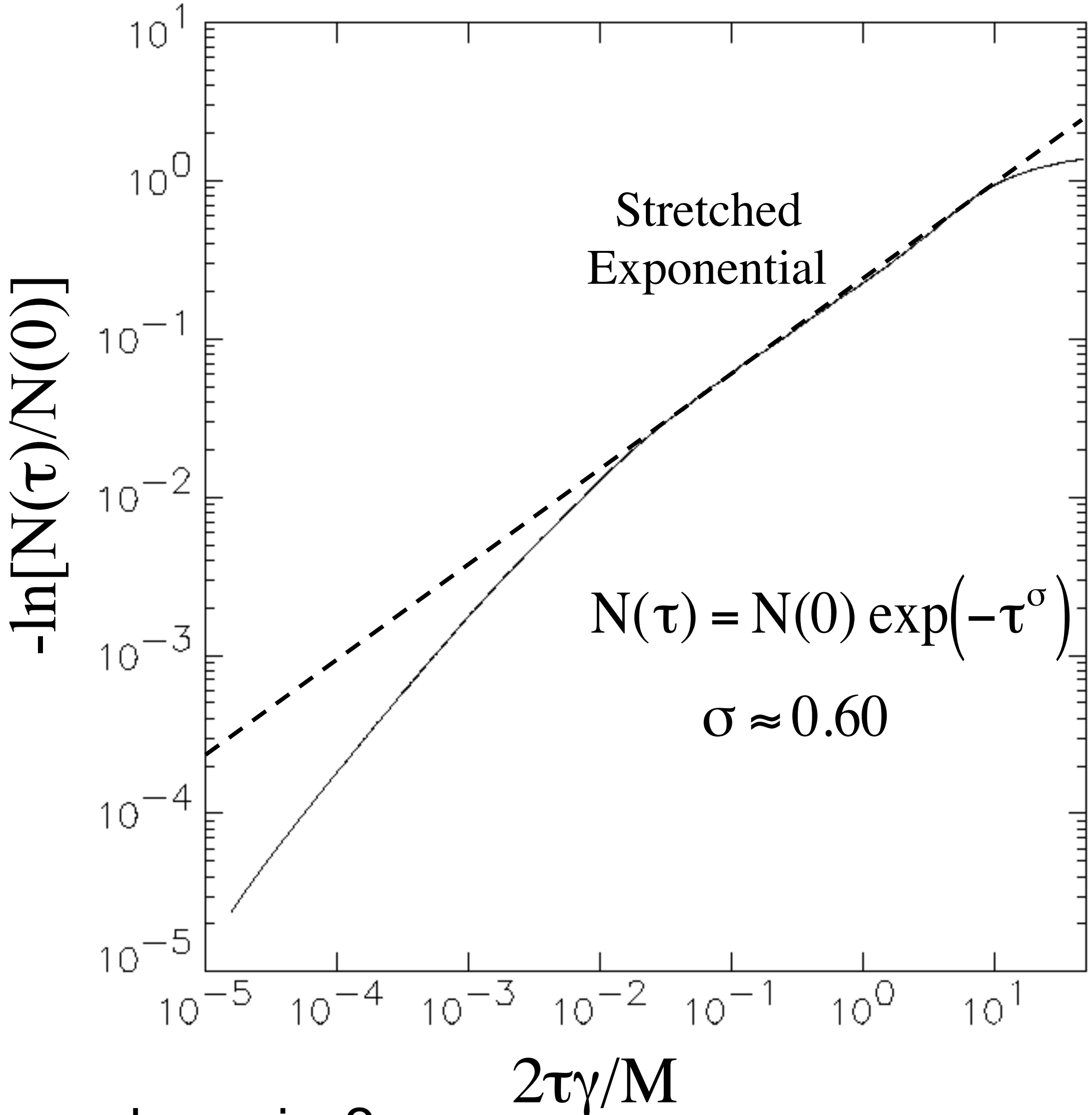


High input power

Localisation: Discrete Breathers, H.S. Eisenberg et al., Phys. Rev. Lett. **81**, 3383 (1998)

Statistics of the Localisation Process

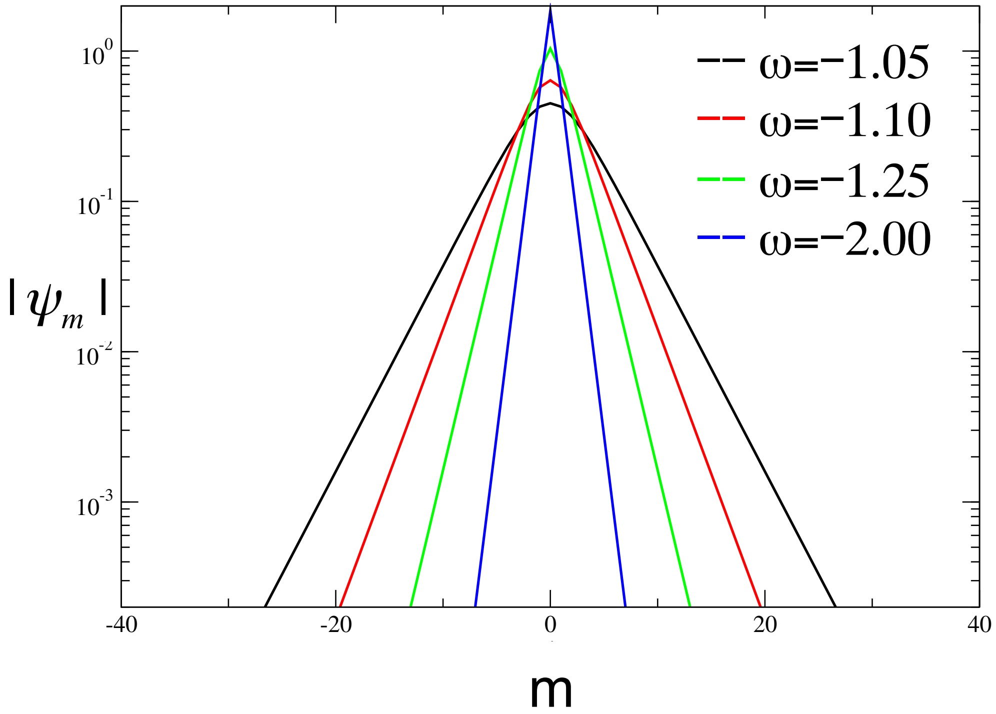




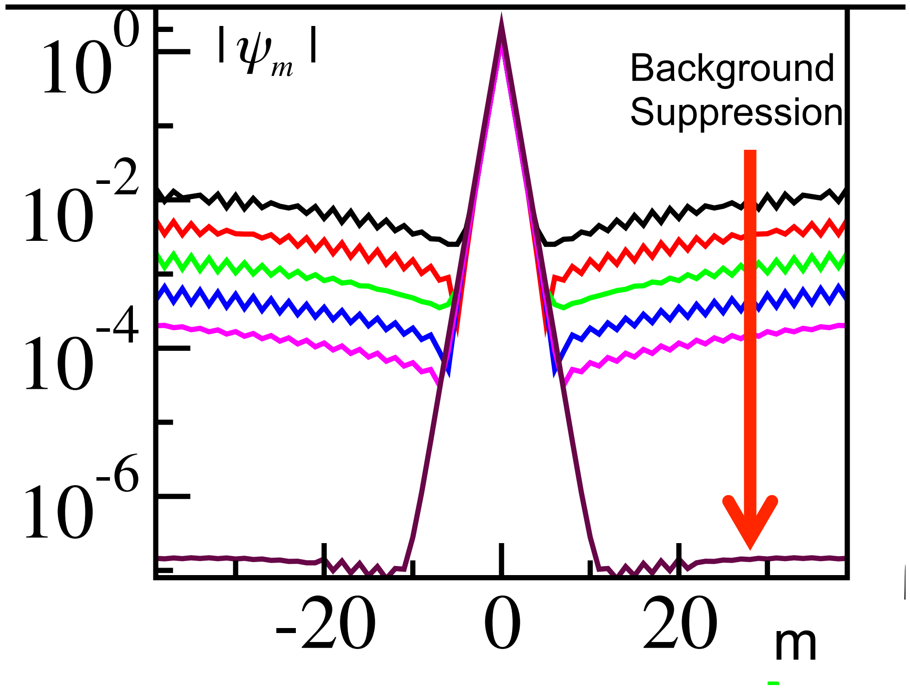
Glassy dynamics?

DNLS: Discrete Breathers, background and boundary losses

Naked Conservative Breathers.
The intensity does not breathe

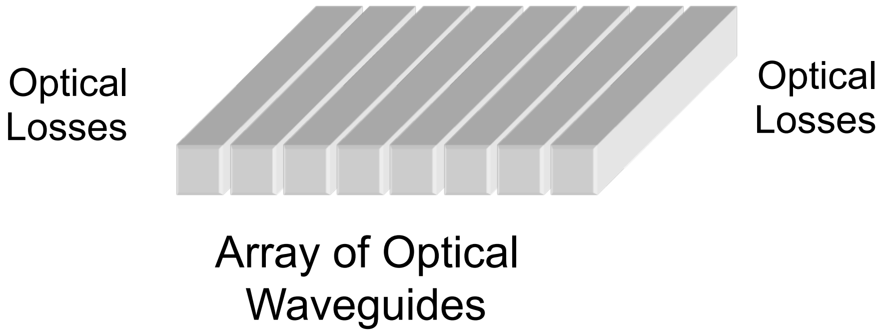


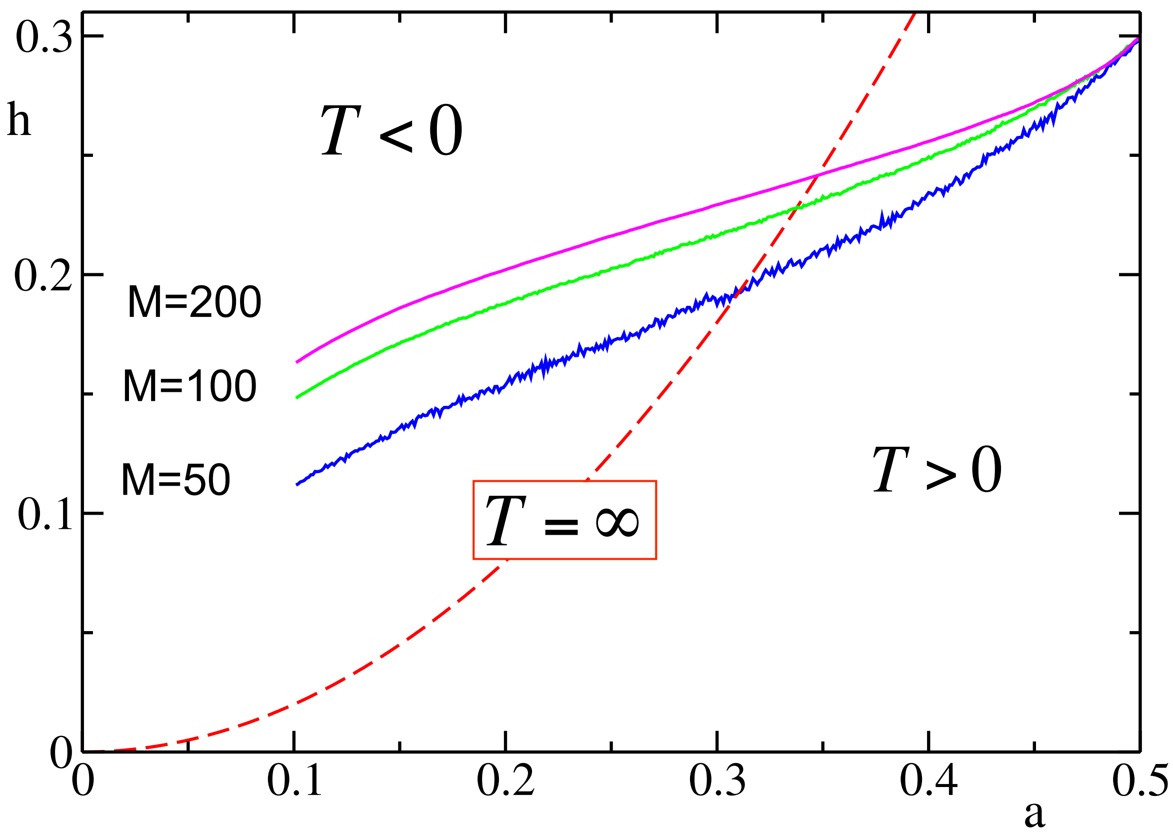
Stripping a Breather Naked
via Boundary Dissipations



For a review see:
R. Franzosi, R. Livi, G-LO, and A. Politi, Nonlinearity **24**, R89 (2011)

How to go from Positive T to Negative T





Yes. We increase T by removing energy and light intensity from the system.

$$\frac{1}{T_\mu} = \left\langle \frac{W \|\vec{\xi}\|}{\vec{\nabla} H \cdot \vec{\xi}} \vec{\nabla} \cdot \left(\frac{\vec{\xi}}{\|\vec{\xi}\| W} \right) \right\rangle_\mu$$

$$\vec{\xi} = \frac{\vec{\nabla} H}{\|\vec{\nabla} H\|} - \frac{\left(\vec{\nabla} H \cdot \vec{\nabla} A\right) \vec{\nabla} A}{\|\vec{\nabla} H\| \|\vec{\nabla} A\|^2}$$

$$W = \left\{ \sum_{\substack{j,k=1 \\ j<k}}^{2M} \left[\frac{\partial H}{\partial x_j} \frac{\partial A}{\partial x_k} - \frac{\partial H}{\partial x_k} \frac{\partial A}{\partial x_j} \right]^2 \right\}^{1/2} .$$

Would it be possible to measure T<0 by measuring output amplitudes and phases?

Is a single Discrete Breather a state at T<0?

Continuous System: The NLS in a Periodic Potential (Lattice) and Lattice Solitons

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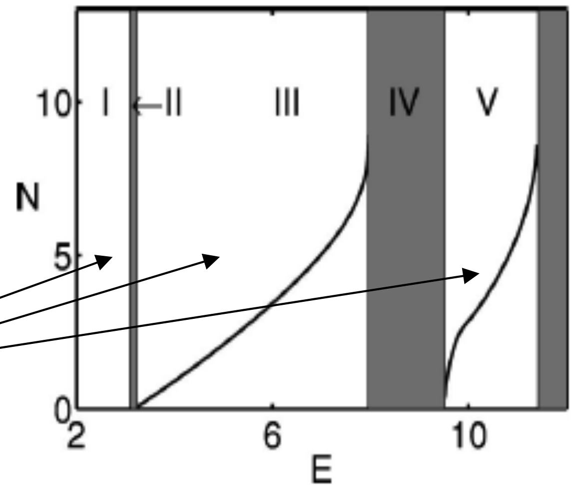
NLS in an OL

$$i\frac{\partial u}{\partial t} = -\frac{1}{2}\frac{\partial^2 u}{\partial x^2} + V_0 \sin^2\left(\frac{\pi x}{2}\right)u + \Lambda |u|^2 u$$

$$N = \int_{-\infty}^{+\infty} |u|^2 dx$$

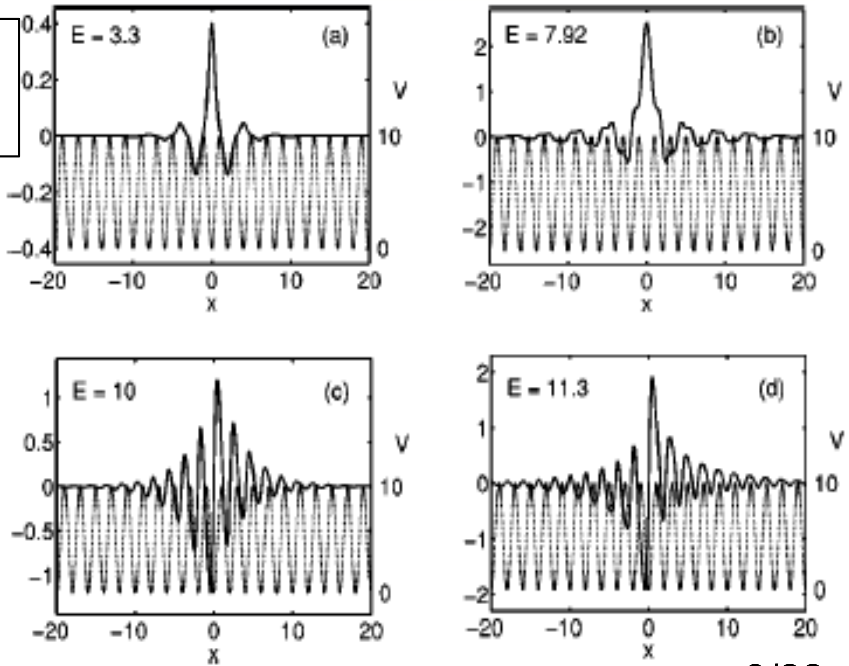
Gaps



Ansatz

$$u(x,t) = v(x)\, e^{-iEt}$$
$$v(x) \Rightarrow e^{-|x|} \;\; for \;\; x \Rightarrow \pm\infty$$

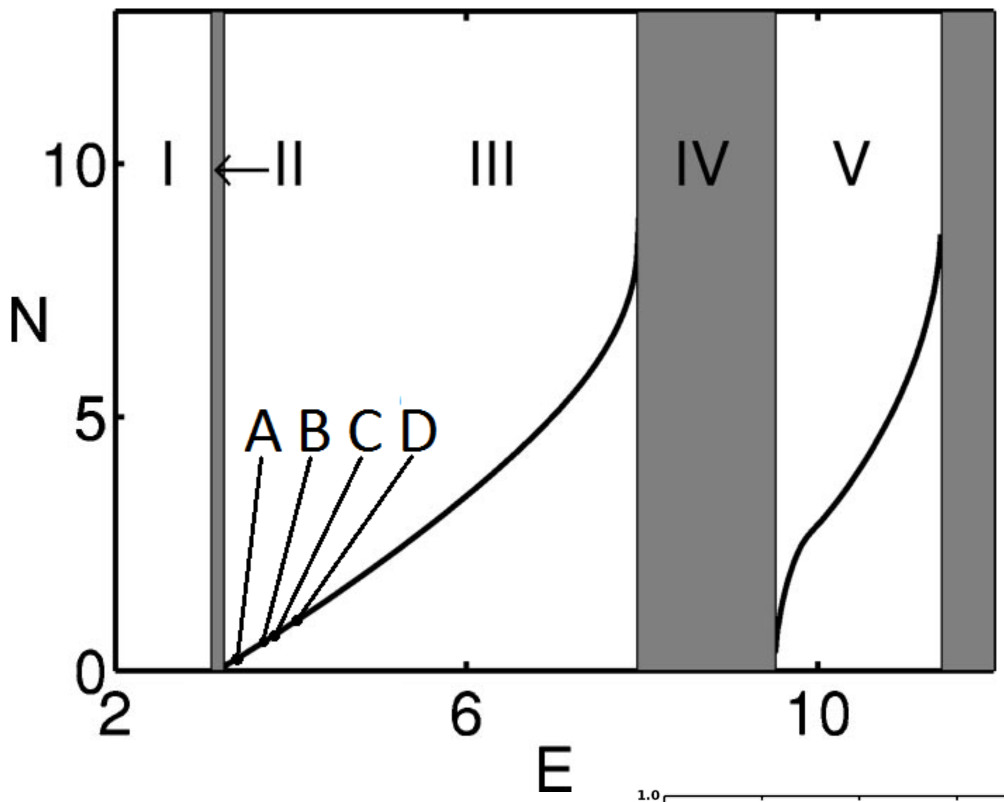
$$Ev = \Lambda v^3 - \frac{1}{2}\frac{\partial^2 v}{\partial x^2} + V_0 \sin^2\left(\frac{\pi x}{2}\right)v$$

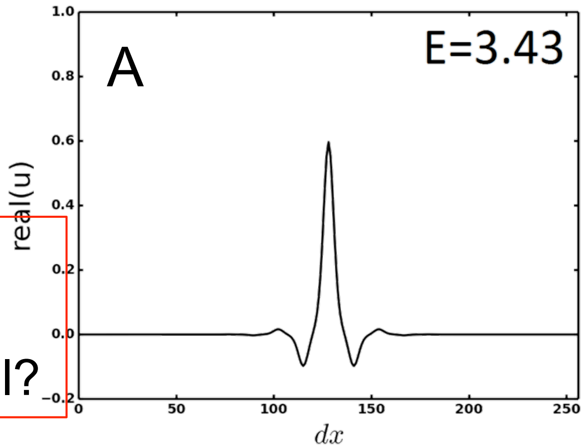


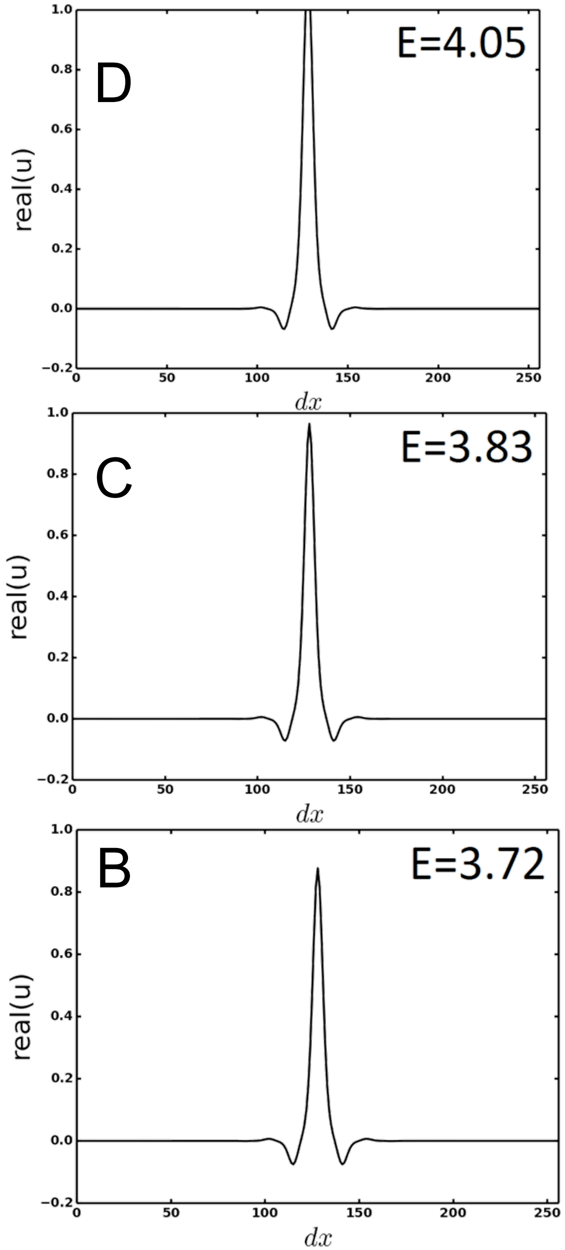
N. Efremidis and D. Christodoulides
Phys. Rev. A 67, 063608 (2003)

Lattice Solitons =
= Continuous Breathers

Lattice Solitons via Boundary Losses

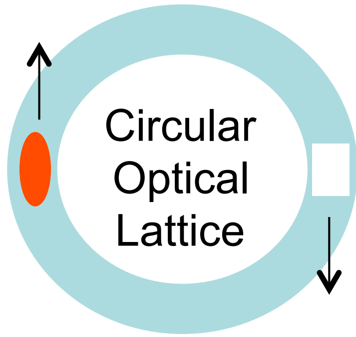


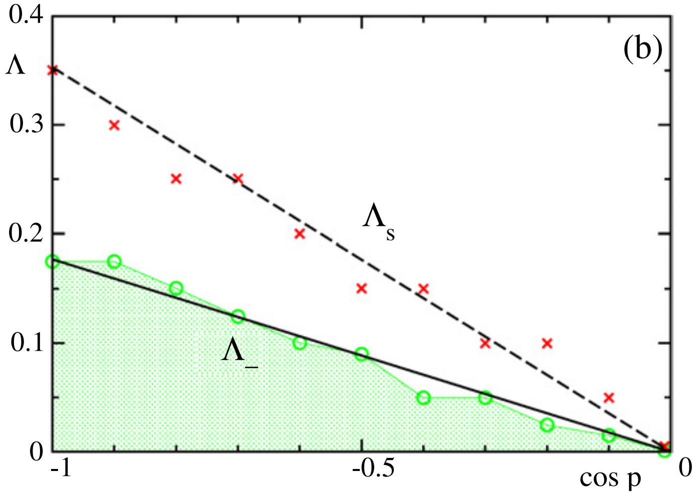




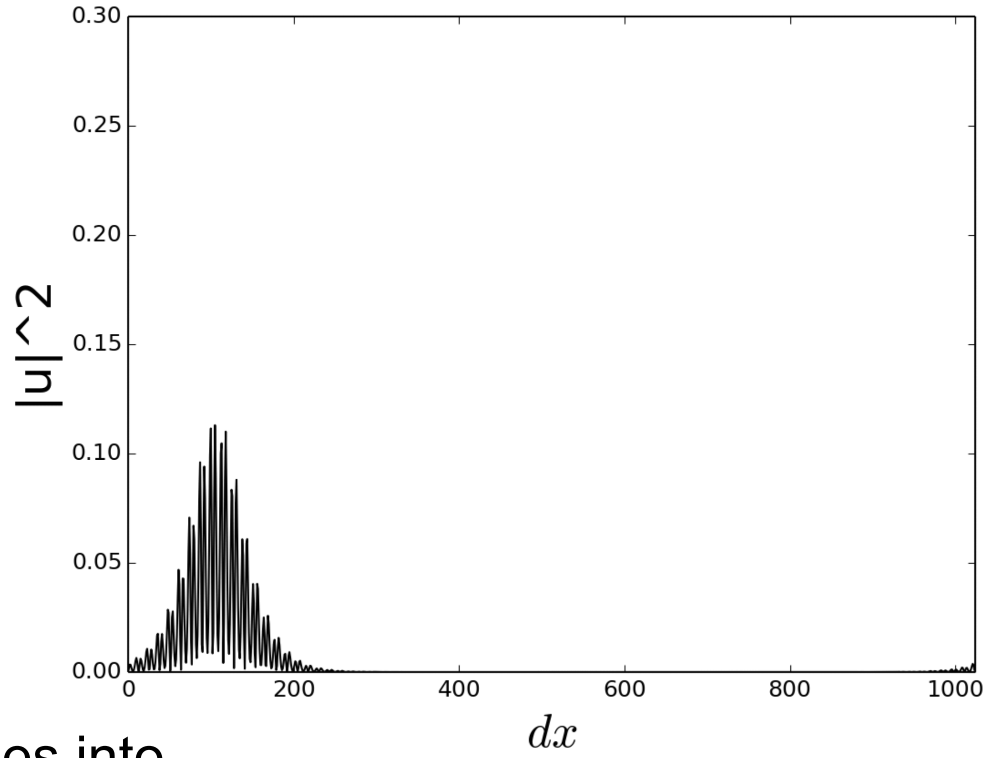
Does this mean moving from Positive to Negative T as in the Discrete Model?

Moving Gap Soliton



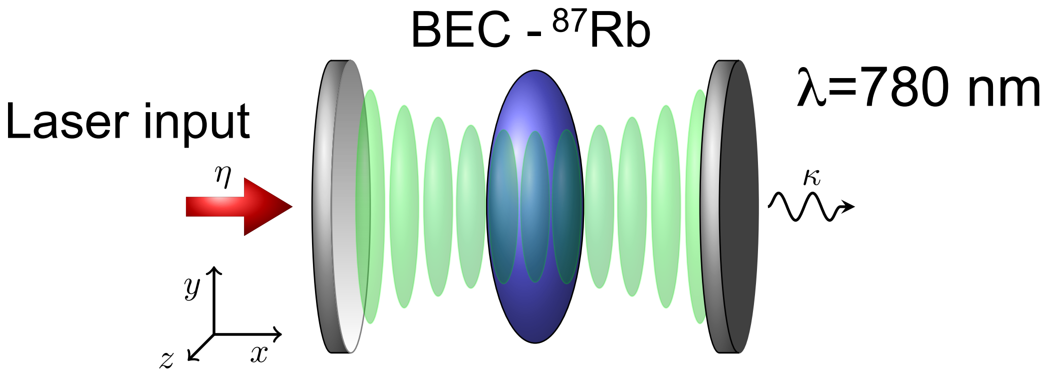


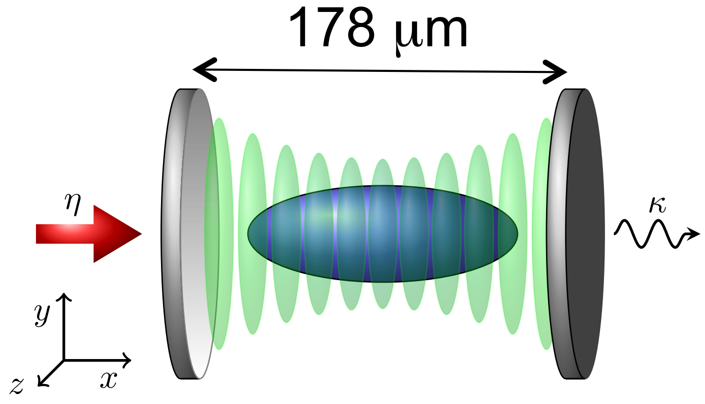
$$\psi(t=0)=\frac{1}{(2\pi)^{1/4}\sqrt{\sigma}}\exp\left(-\frac{(m-\hat{m})^2}{4\sigma^2}+ip(m-\hat{m})\right)$$



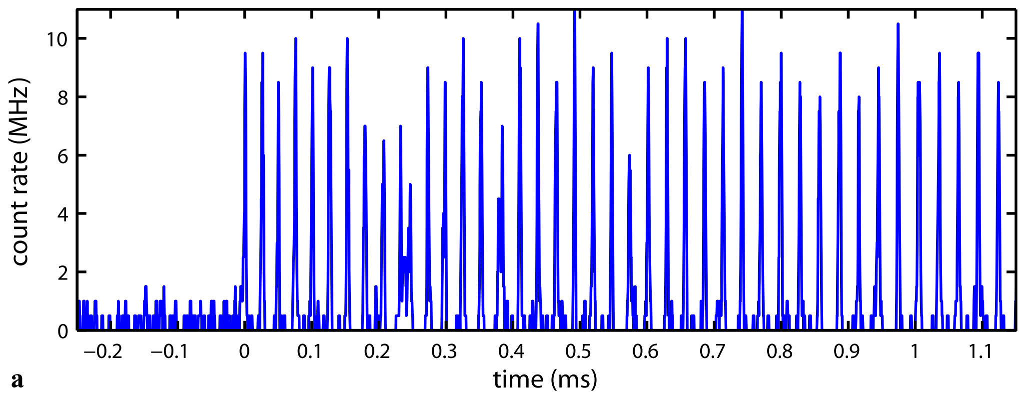
The Gaussian Wavepacket reshapes into the gap soliton. The 'debris' are cleaned by the travelling losses.

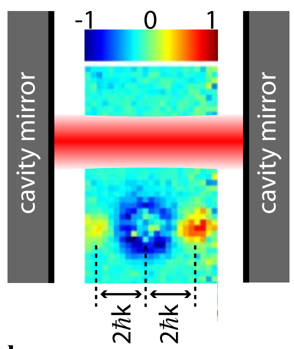
Superfluid BEC in an Optical Cavity





Non-equilibrium process: Externally driven system with cavity losses



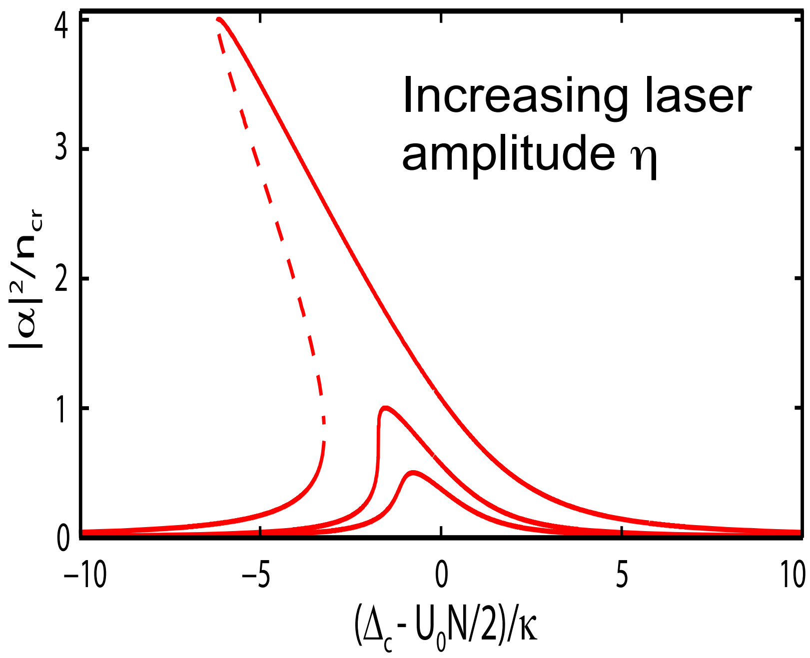


Oscillations when laser is on. Atomic motion too. Gorgeous stuff

F. Brennecke et al. Science **322**, 235 (2008)
S. Ritter et al., App. Phys. B **95**, 213 (2009)

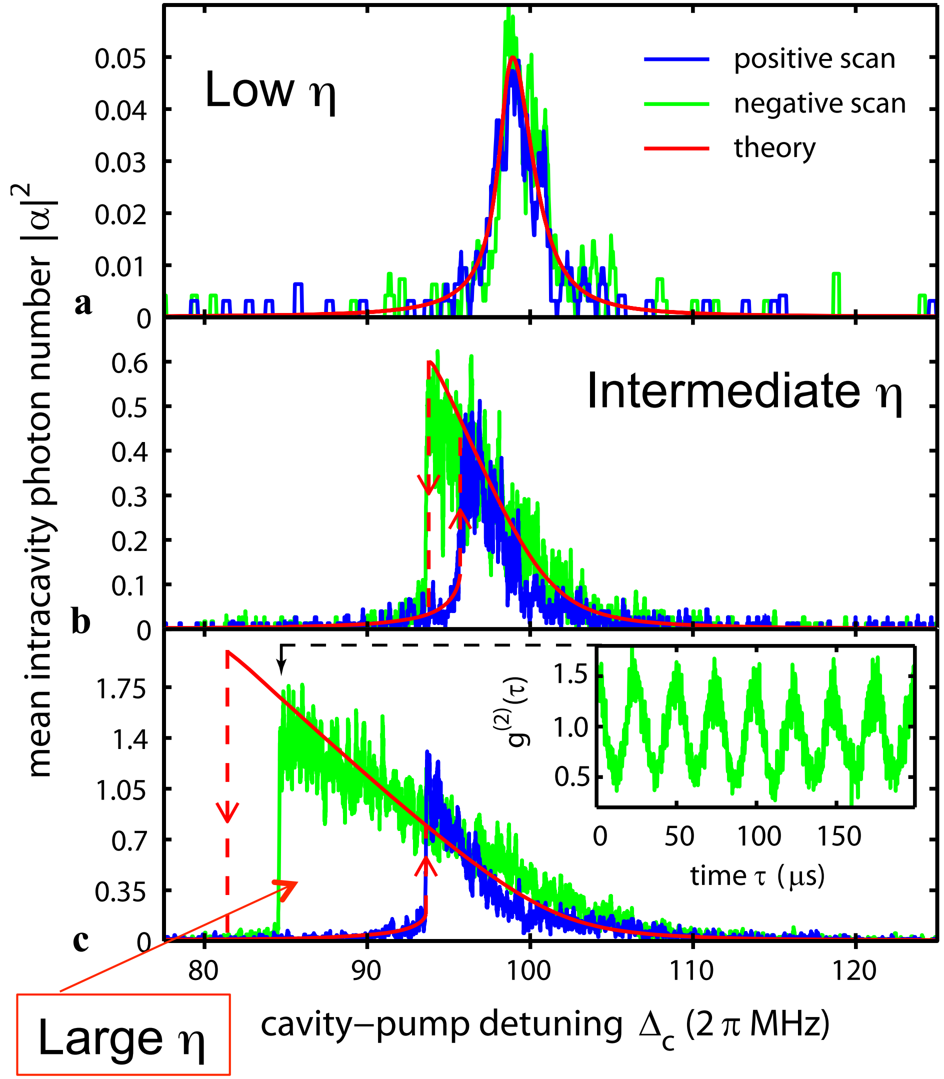
Tilman Esslinger’s group
ETH, Zurich, Switzerland

Bistability



Theory (see later)

Experiment



The model (Horak, Barnett, Ritsch, PRA 2000)

$$\frac{\partial\psi(x,t)}{\partial t} = \frac{i}{2}\frac{\partial^2\psi}{\partial x^2} - i|\alpha|^2 V_0 \sin^2\left(\frac{\pi x}{2}\right)\psi - i\Lambda|\psi|^2\psi$$

BEC wave-function
Λ atom-atom interact.

$$\frac{\partial\alpha(t)}{\partial t} = i\Delta_c\alpha - iU_0N\langle\psi|\sin^2(\pi x/2)|\psi\rangle\alpha - \kappa\alpha + \eta$$

Cavity field
$N \sim 10^5 - 10^6$

Cavity-pump detuning

Overlap cavity and BEC modes

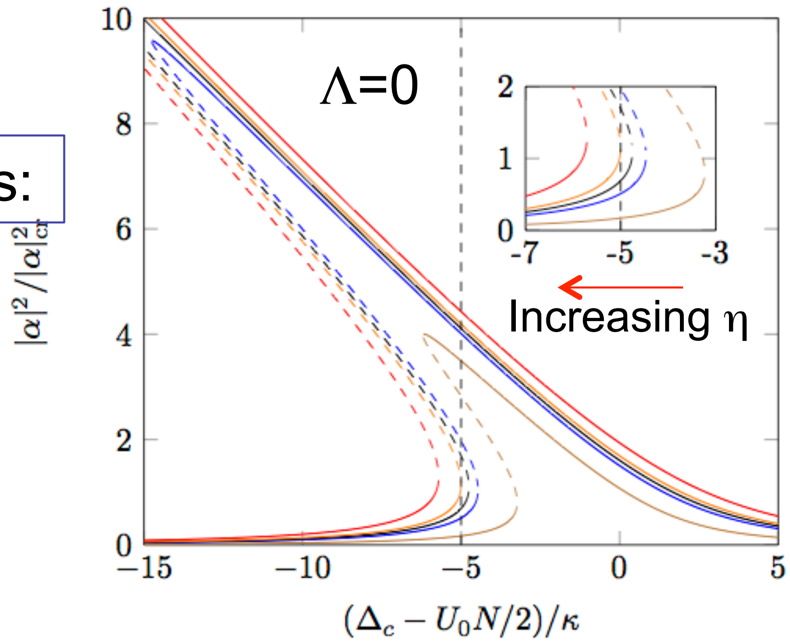
Losses

Pump

By eliminating the fast optical field one gets:

$$|\alpha|^2 = \frac{\eta^2}{\kappa^2 + \left(\Delta_c - U_0N\langle\psi|\sin^2(\pi x/2)|\psi\rangle\right)^2}$$

$$U_0 = g_0^2/\Delta_a \qquad V_0 = U_0T = U_0m(\lambda/2)^2/(4\hbar)$$



Conservative or dissipative dynamics?

$$\frac{\partial\psi(x,t)}{\partial t}=\frac{i}{2}\frac{\partial^2\psi}{\partial x^2}-i|\alpha|^2 V_0\sin^2\left(\frac{\pi x}{2}\right)\psi-i\Lambda|\psi|^2\psi$$

$$|\alpha|^2=\eta^2/\left[\kappa^2+\left(\Delta_c-U_0N\langle\psi|\sin^2(\pi x/2)|\psi\rangle\right)^2\right]$$

- Losses are losses so from a generic initial condition the system dissipates
- However, dissipations end when losses κ are balanced by pump η
- That happens when the field follows the BEC wave-function (on the center manifold). Here the flow is symplectic since for the Jacobian *J* one has:

$$SJ+J^TS=0 \qquad S=\begin{bmatrix}0 & I\\ -I & S\end{bmatrix}$$

Hamiltonain flows are symplectic
Symplectic flows maybe Hamiltonian
Symplectic flows <u>ARE</u> conservative

Conservative Dynamics

So?

Numerical Integration ($\Lambda = 0$)

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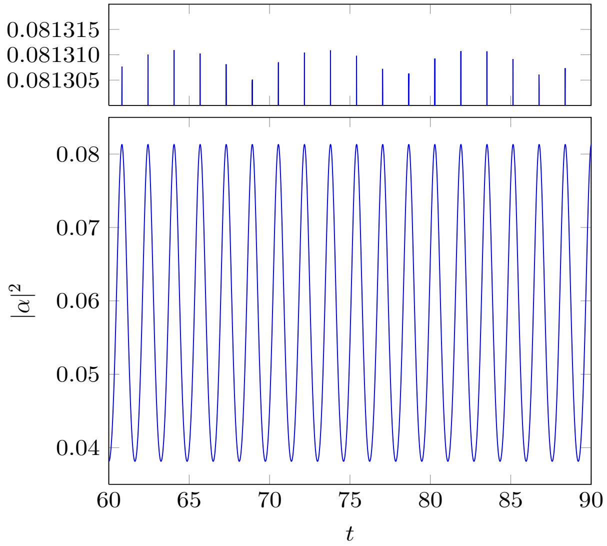
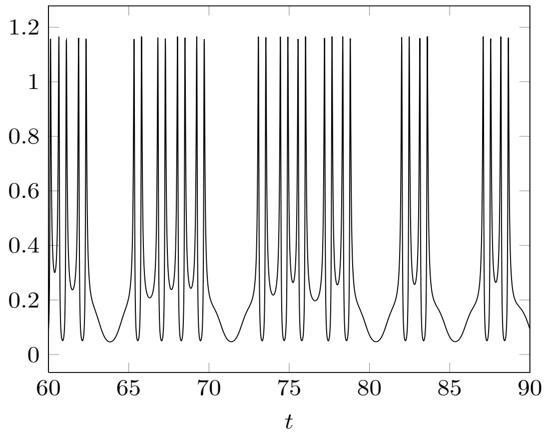
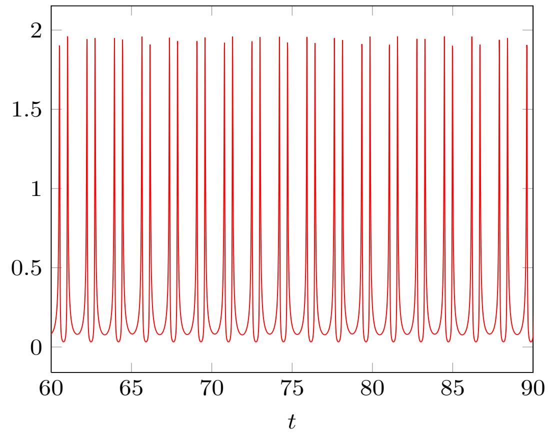
Chaos near Separatrix



(a) $\eta/\kappa = 0.99$



(b) $\eta/\kappa = 1.08$

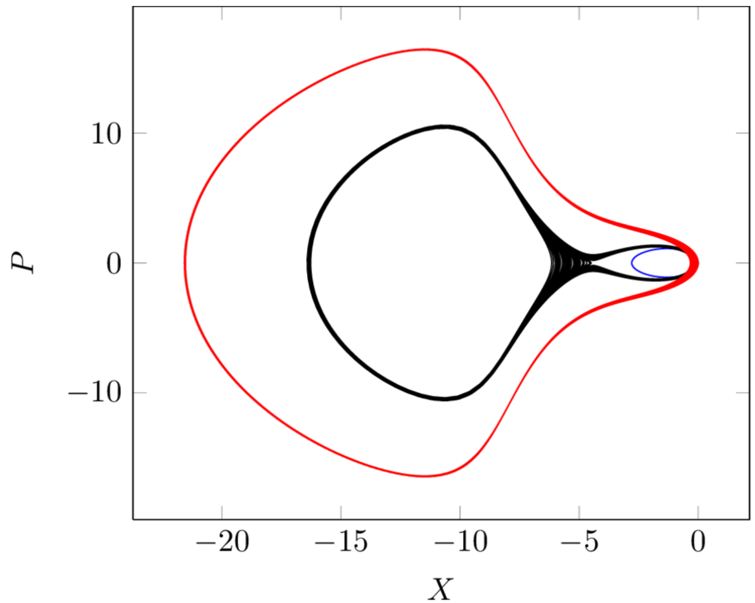


(c) $\eta/\kappa = 1.40$

$$c_n(t) = \frac{1}{L_D} \int_{-L_D/2}^{L_D/2} \psi(x,t)\, e^{-i\pi n x}\, dx$$

$$X \propto -\mathrm{Re}\left[c_0 (c_1^* + c_{-1}^*)/2\right]$$

$$P \propto \mathrm{Im}\left[c_0 (c_1^* + c_{-1}^*)/2\right]$$

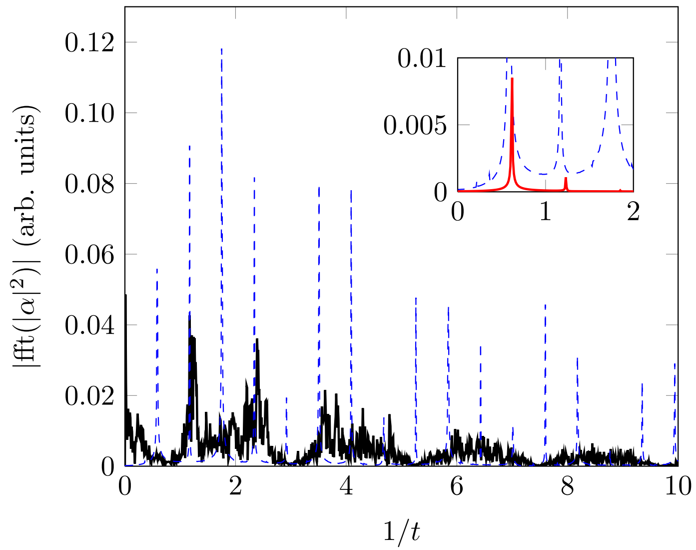


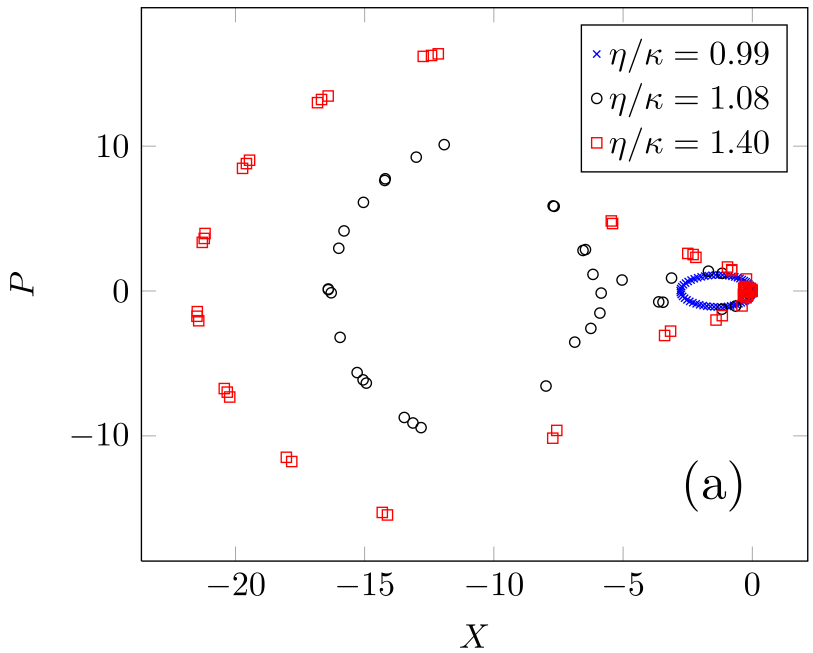
P = Momentum

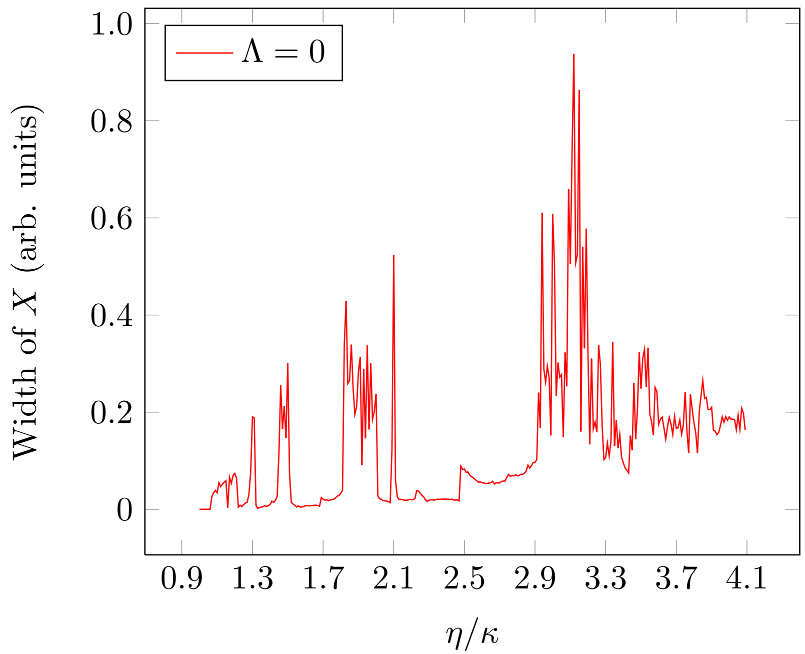
Is it really conservative chaos?

- Experimental initial condition: all atoms in c_0, flat wave-function.
- Dynamics depends on initial condition.
- Experimental initial condition self-consistent with model (mean field).
- Spectra, Poincare' sections, thickness of Poincare' sections

YES



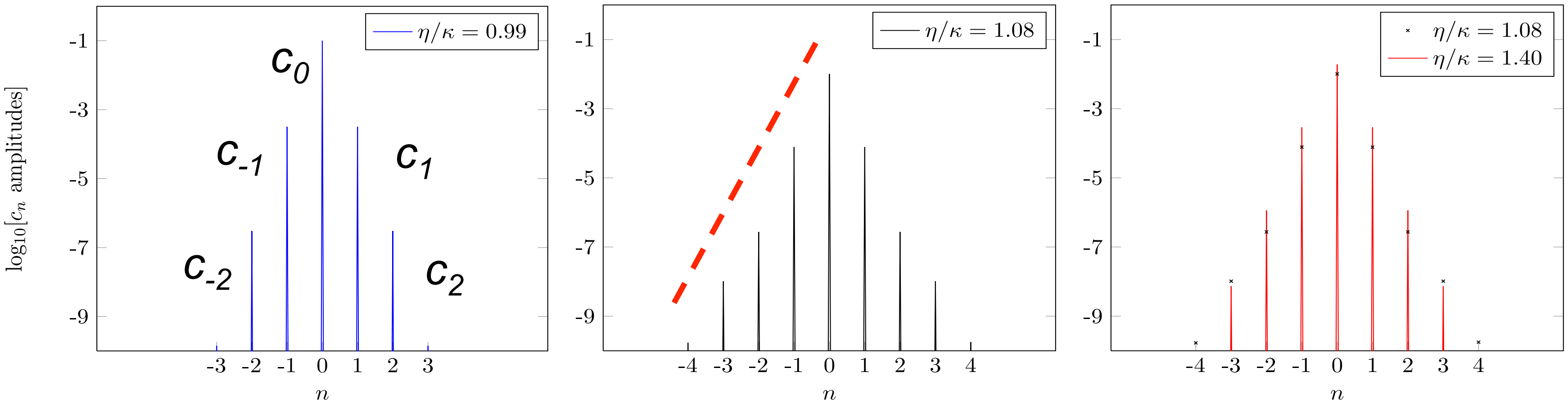




thickness of Poincare' section

Modal Expansion

$$c_n(t) = \frac{1}{L_D}\int_{-L_D/2}^{L_D/2} \psi(x,t)\,\mathrm{e}^{-\mathrm{i}\pi n x}\,\mathrm{d}x,$$



(a) $\eta/\kappa = 0.99$ (b) $\eta/\kappa = 1.08$ (c) $\eta/\kappa = 1.40$

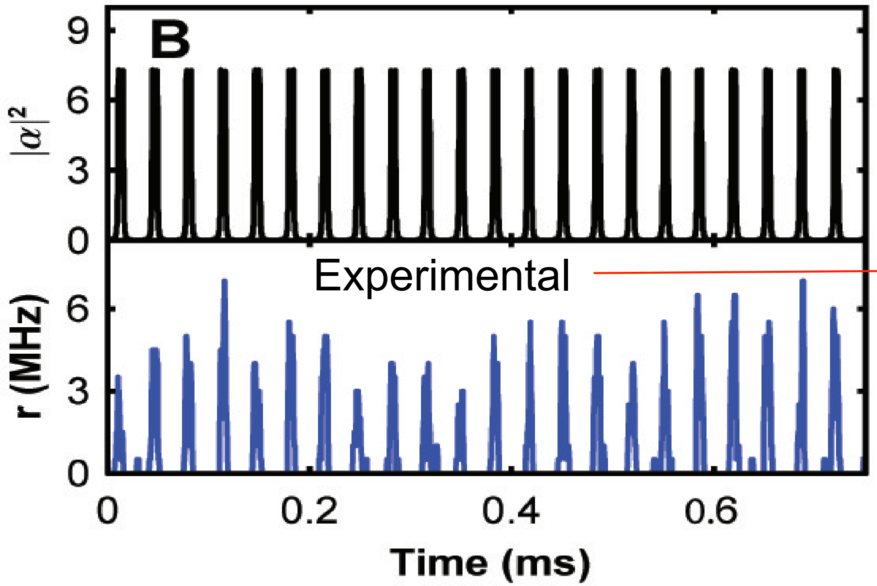
Log_{10} of modal amplitudes

$$\psi(x,t) = \frac{1}{\sqrt{n_L}}\sum_{n=-N_{\max}}^{N_{\max}} c_n(t)\mathrm{e}^{\mathrm{i}n\pi x}, \qquad \left\langle\psi\middle|\sin^2\left(\frac{\pi x}{2}\right)\middle|\psi\right\rangle = \frac{1}{2}\left[1 - \sum_{n=-N_{\max}}^{N_{\max}} c_n^*\left(c_{n-1}+c_{n+1}\right)\right]$$

$$\frac{dc_n}{dt} = -\mathrm{i}\frac{n^2\pi^2}{2}c_n - \mathrm{i}\frac{|\alpha|^2 V_0}{2}\left(c_n - \frac{c_{n-1}}{2} - \frac{c_{n+1}}{2}\right)$$
$$-\mathrm{i}\Lambda' \sum_{k,m=-N_{\max}}^{N_{\max}} c_k c_m^* c_{n+m-k}$$

How many modes?

Comparison with experiments



Two or three modes theory.

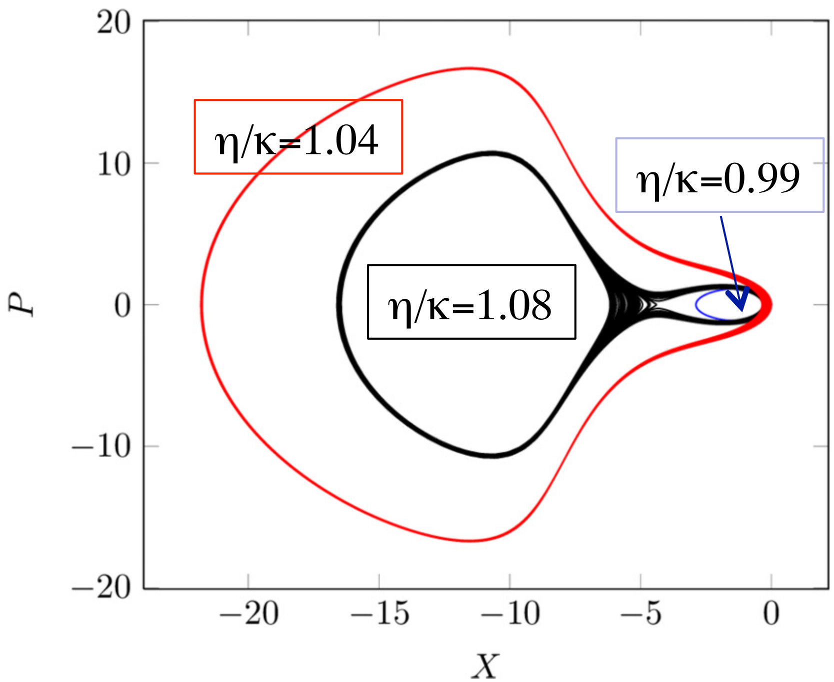
F. Brennecke et al. Science **322**, 235 (2008)

The time scale of the oscillations is predicted correctly <u>but</u> the nature of the oscillations is not correct. In the experiments the fluctuations were 'explained' by a dodgy detector (we do not think so).

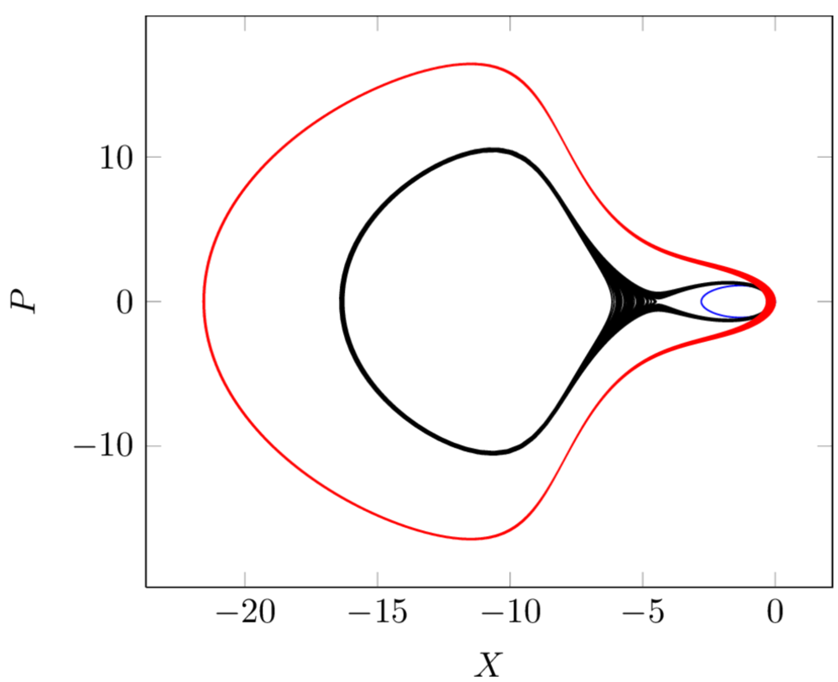
Although the modal amplitude decreases exponentially with the mode index, more modes are NECESSARY to explain the experiments

Five mode model

- 5 Modes c_0, c_1, c_{-1}, c_2, c_{-2} but only 5 Independent Variables



Five equation ODE model



Full PDE model

Impressive. It includes chaos

Inclusion of atom-atom interactions (Λ>0)

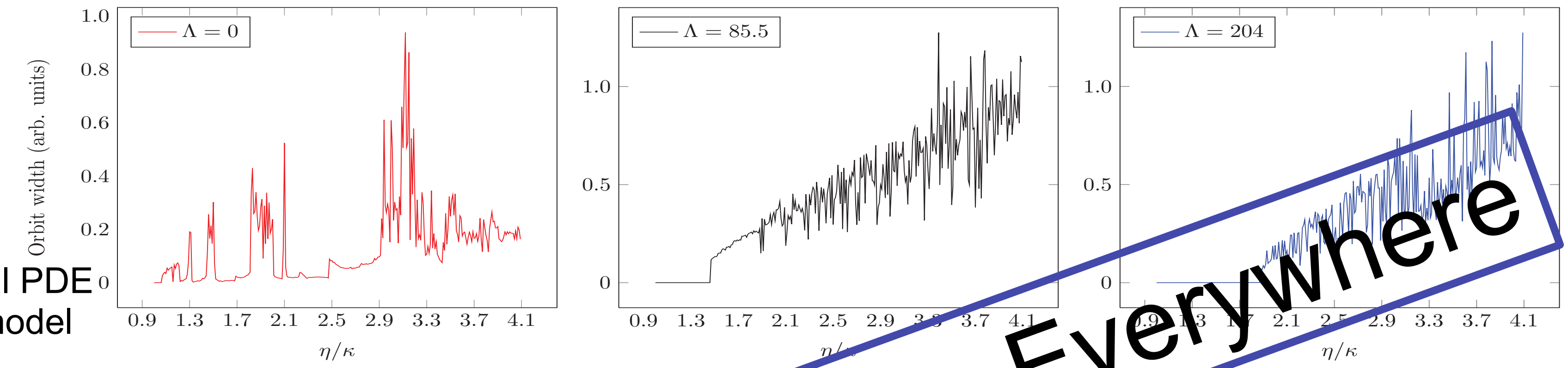
$$\frac{\partial \psi(x,t)}{\partial t} = \frac{i}{2}\frac{\partial^2 \psi}{\partial x^2} - i|\alpha|^2 V_0 \sin^2\left(\frac{\pi x}{2}\right)\psi - i\Lambda|\psi|^2\psi$$

$$|\alpha|^2 = \eta^2 / \left[\kappa^2 + \left(\Delta_c - U_0 N \langle\psi|\sin^2(\pi x/2)|\psi\rangle\right)^2\right]$$

The usual nonlinearity

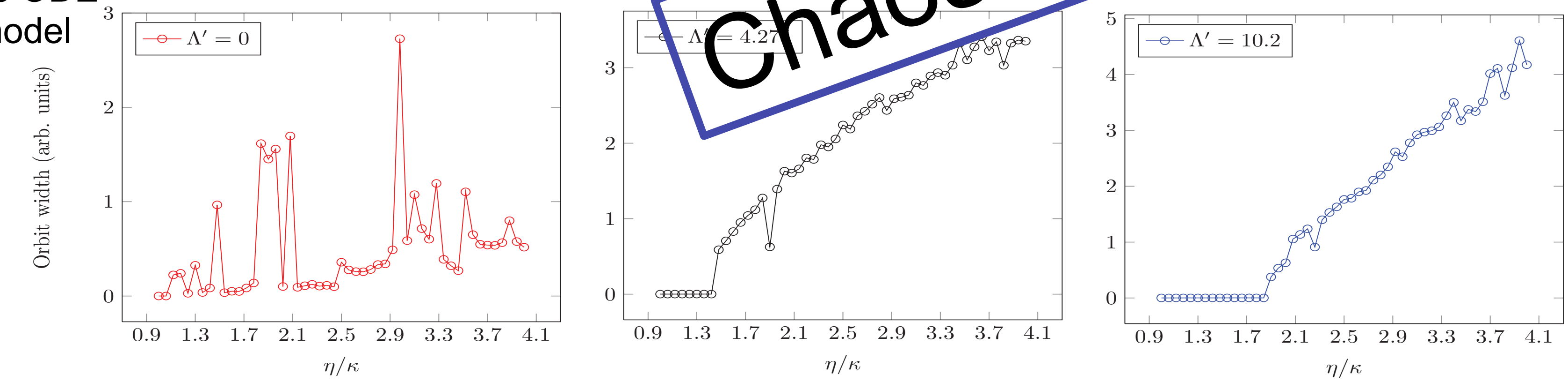
Thickness of Poincare' sections

Full PDE model

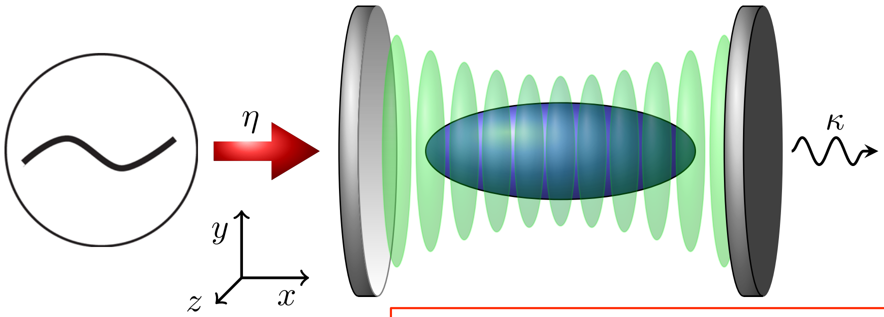


(a) Λ = 0 (b) Λ = 85.5 (c) Λ = 204

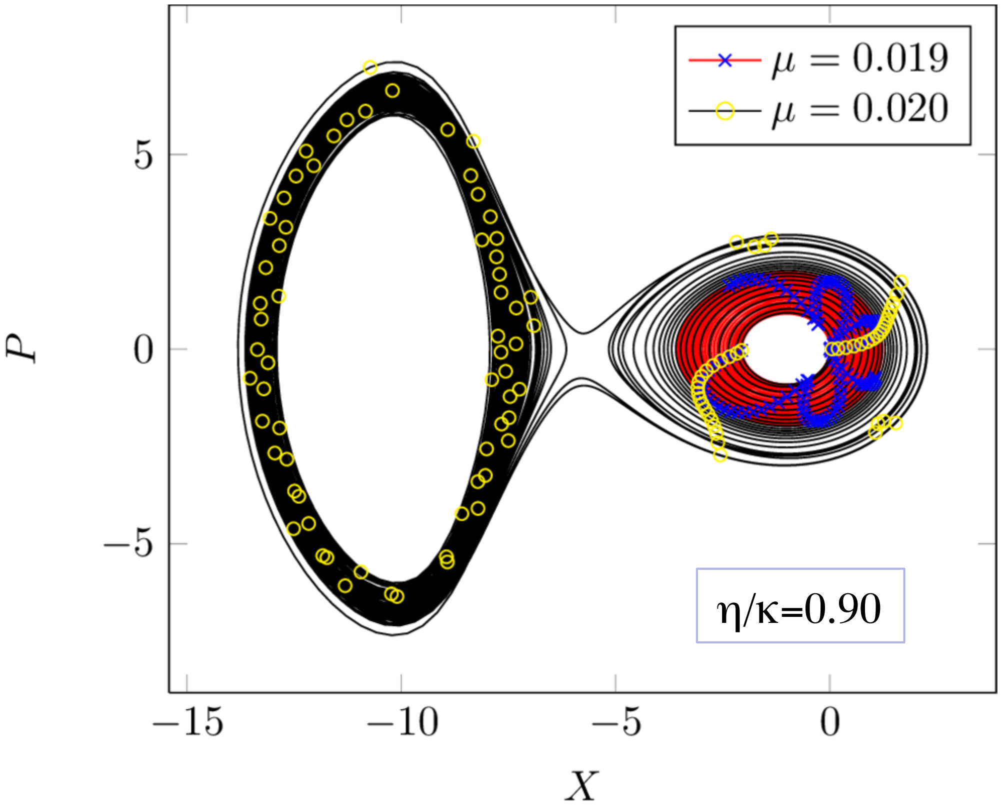
Five ODE model

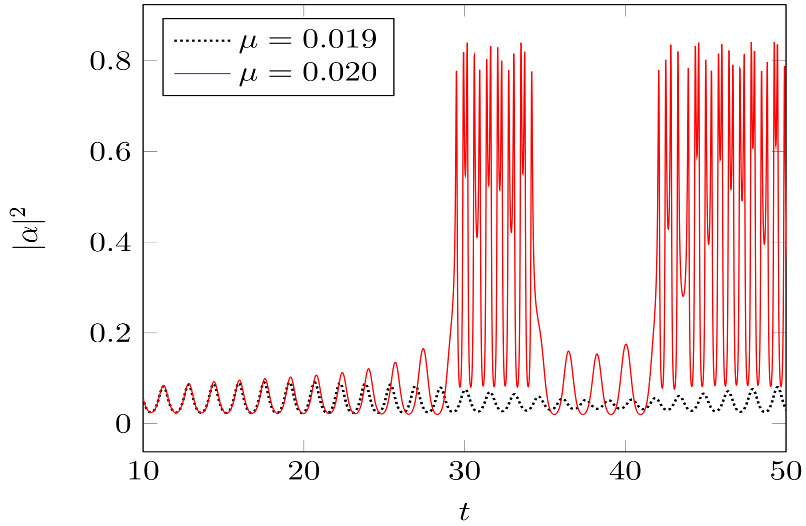


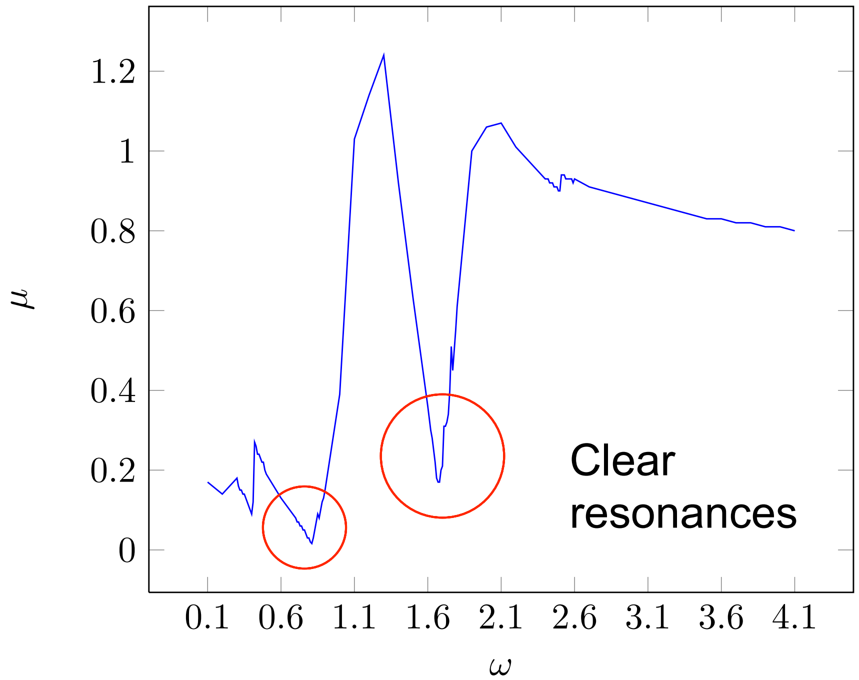
Modulating the input laser amplitude



$$\eta \quad \Rightarrow \quad \eta\,[1+\mu\cos(\omega t)]$$







Conclusions

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✓ Continuous models reproduce the localization process seen in the DNLS.

✓ Moving from positive T to negative T? (Statistical analysis still missing)

✓ Next. BEC in optical cavities are an interesting mixture of conservative (BEC) and dissipative (optical cavity) dynamics

✓ Conservative chaotic motion is the norm when atom-atom interactions are included

✓ Inclusion of the dynamics of the optical field. Multi-longitudinal optical modes.
✓ Mass conserved. Energy flow conservative. Anomalous behaviour?

✓ Longitudinal lattice solitons?

For more details see:
M. Diver, G.R.M. Robb and G.-L. O., Phys. Rev. A **89**, 033602 (2014)