

# Coupled photonics waveguides and cavities

Alessia Pasquazi

*Dept. of Physics and Astronomy, University of Sussex, Brighton BN1 9QH UK*

[a.pasquazi@sussex.ac.uk](mailto:a.pasquazi@sussex.ac.uk)

# Outline



***The DNLSE in optics***

***Array of waveguides-Spatial lattices***

***purely nonlinear lattices***

***Synthetic optical lattices –temporal lattices***

***Optical microresonators and optical frequency combs***

***Motivation***

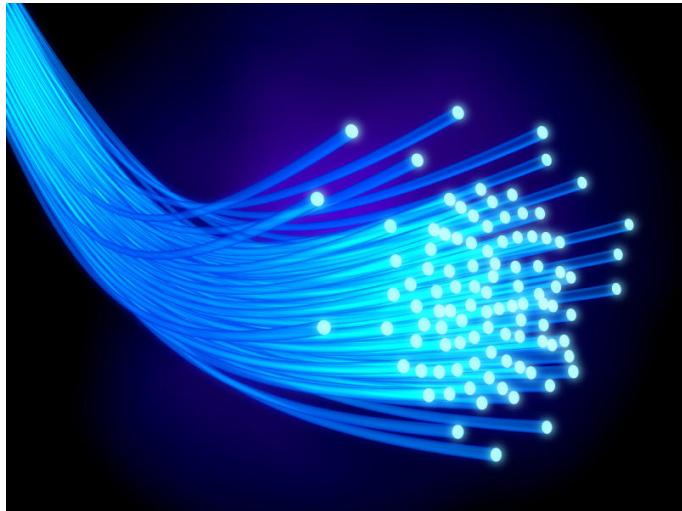
***Low power and large bandwidth***

***The question of coherence***

***Nested cavities approach***

***Efficient modelling of the system***

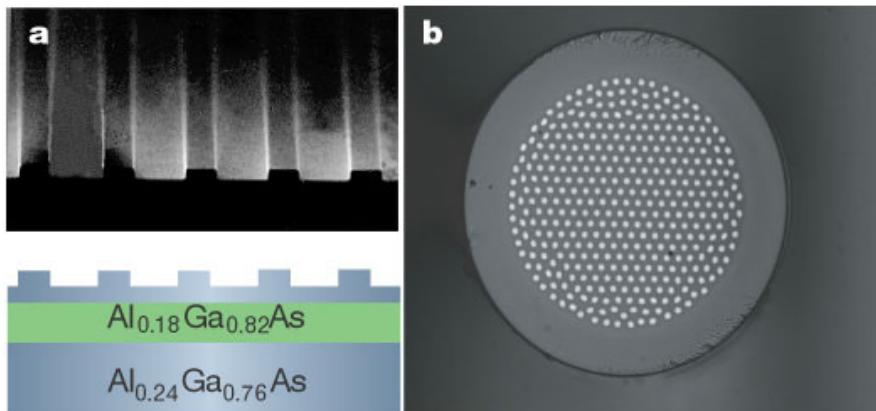
## *.. Recipe n 1 to build a DNLSE*



- 1) Take a bunch of waveguides
- 2) Put them close
- 3) Add some power, nonlinearity is there
- 4) Watch the spatial propagation of the light

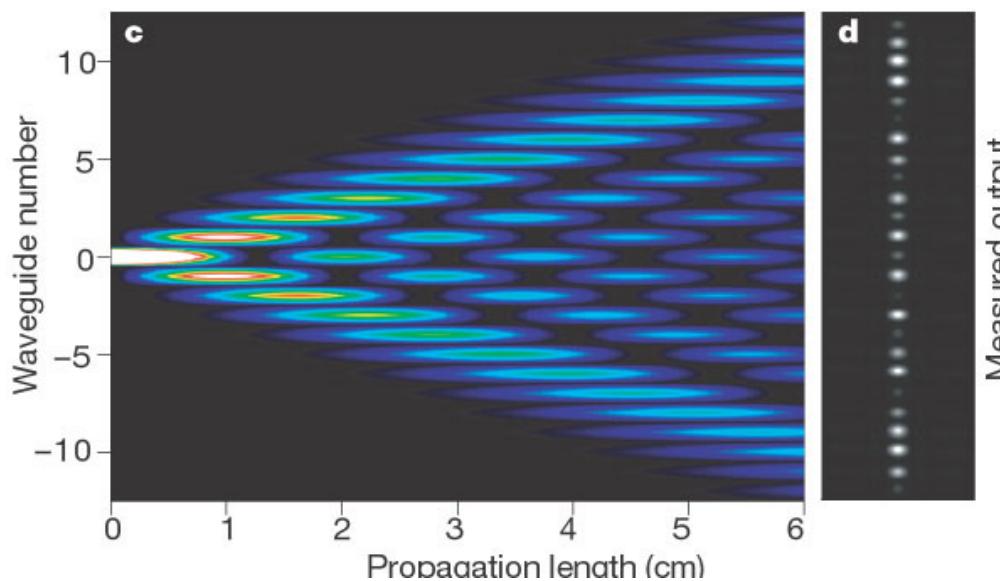
$$i\partial_z u_n(z) = k u_n(z) + \Gamma[u_{n+1}(z) + u_{n-1}(z)] + \gamma|u_n(z)|^2 u_n(z)$$

# *Discrete Array of Waveguides*



Aluminium gallium arsenide (AlGaAs)  
arrays (University of Glasgow, UK)

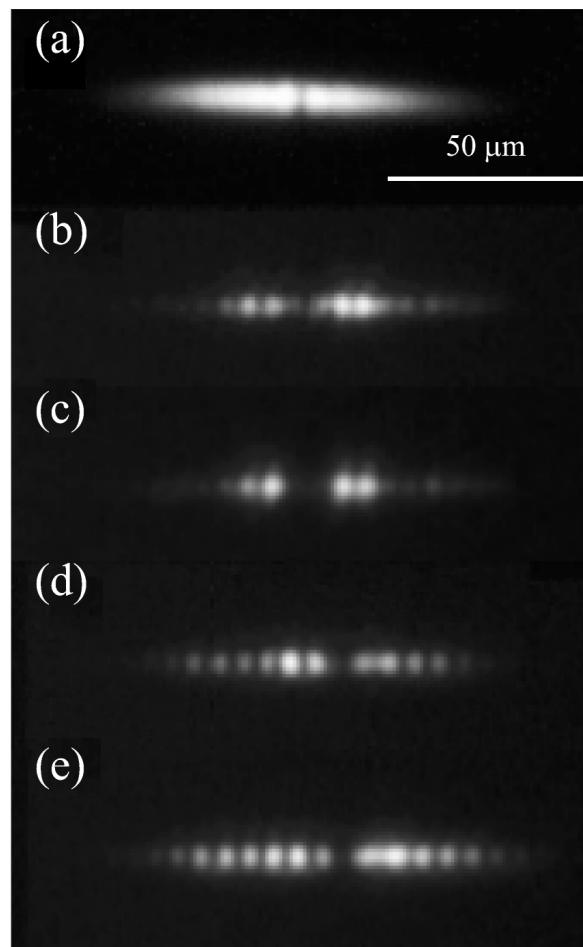
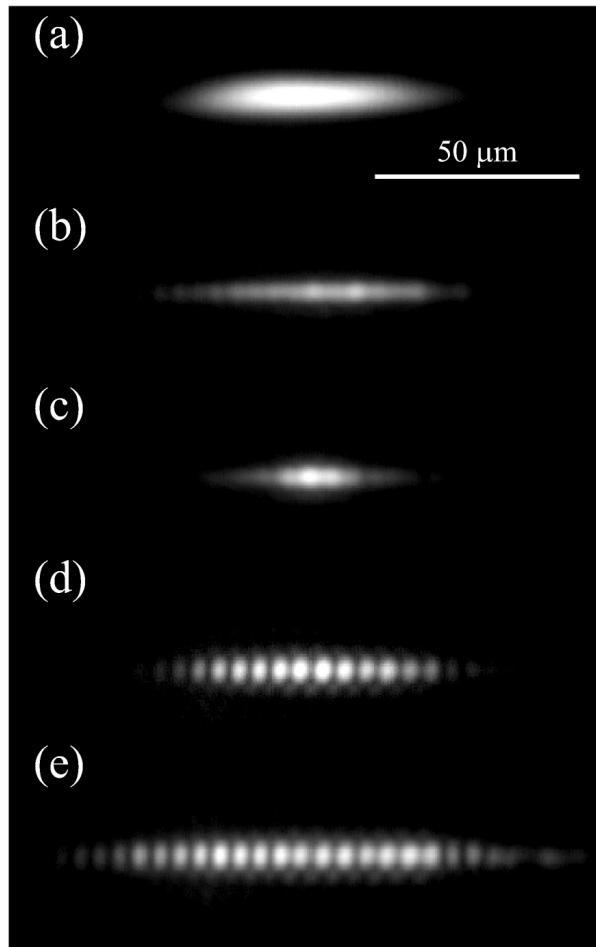
Silica glass (Institute for Physical High  
Technology Jena, Germany)



"Discrete solitons in optics" F. Lederer, G.I. Stegeman, D.N. Christodoulides, G Assanto, M. Segev and Y. Silberberg, Phys. Rep. 463, 1-126 (2008).

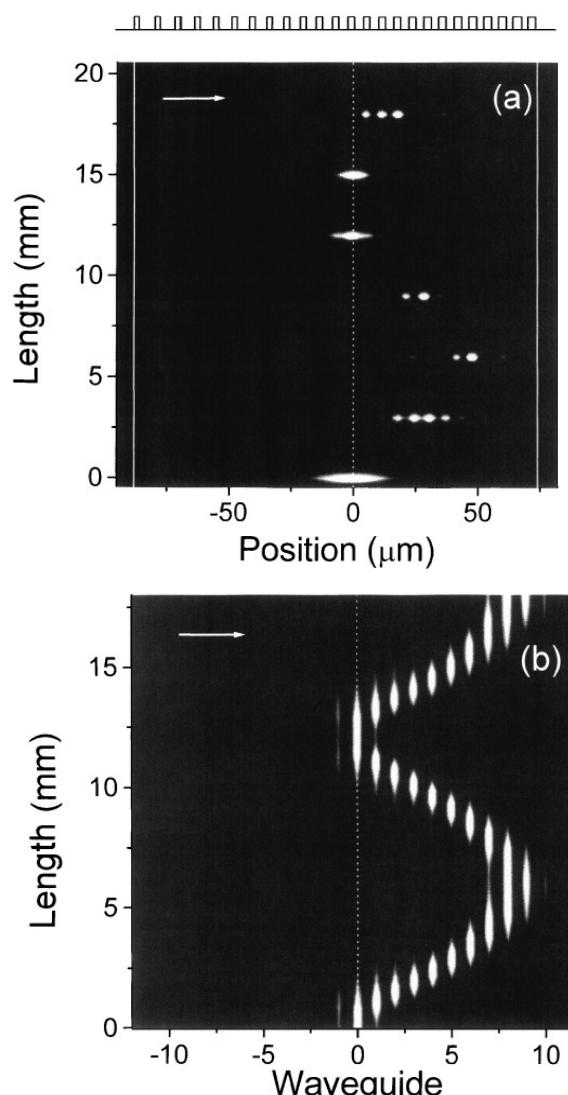
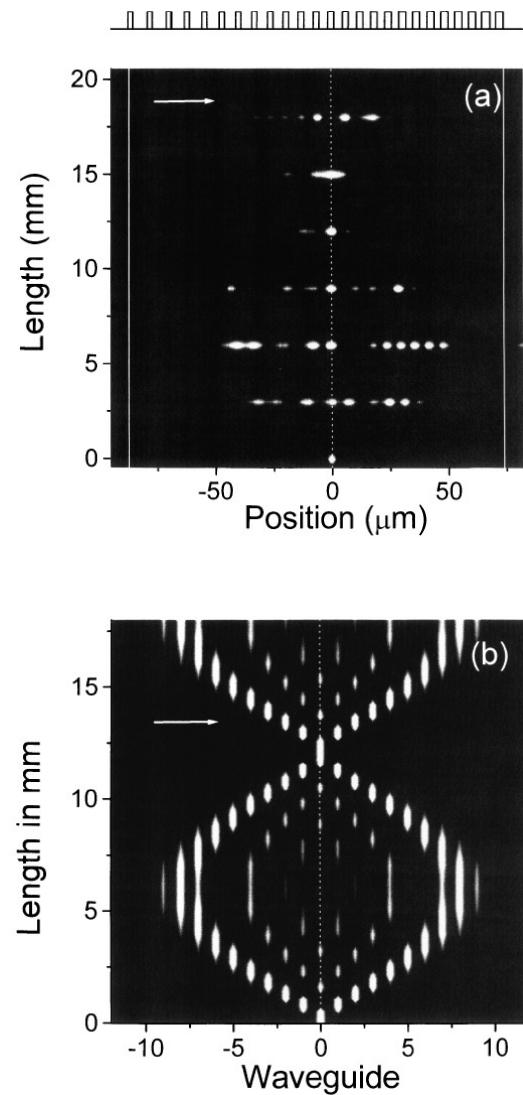
"Discretizing light behaviour in linear and nonlinear waveguide lattices", D. Christodoulides, F. Lederer and Y. Silberberg, Nature, 424, 817 (2003).

# *Bright and Dark Solitons*



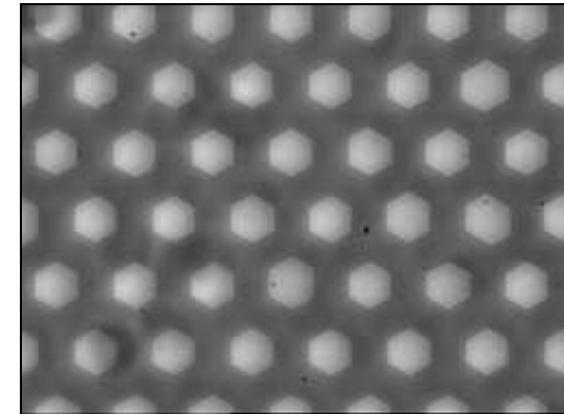
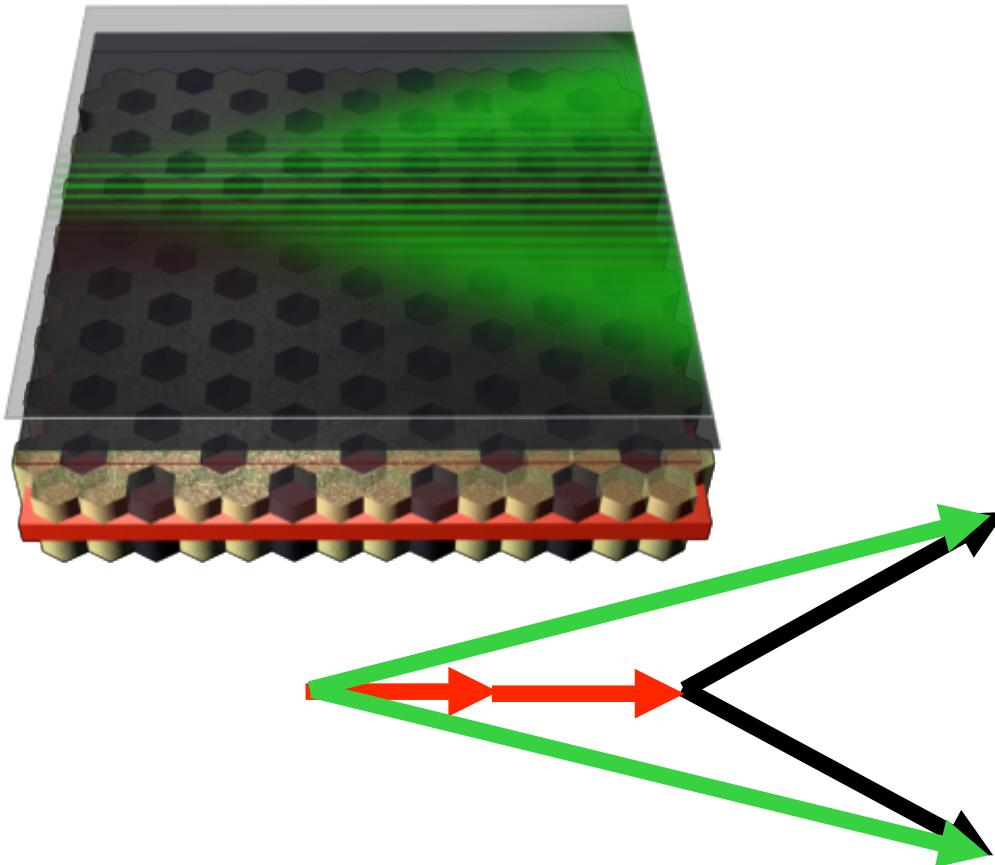
*"Self-Focusing and Defocusing in Waveguide Arrays",*  
R. Morandotti, H. S. Eisenberg, Y. Silberberg, M. Sorel  
and J. S. Aitchison, Phys. Rev. Lett. **86**, 3296 (2001).

# Bloch Oscillations



*"Experimental Observation of Linear and Nonlinear Optical Bloch Oscillations"*, R. Morandotti, U. Peschel, J.S. Aitchison, H. Eisenberg and Y. Silberberg, Phys. Rev. Lett., 83, 4756 (1999).

# HexLN: a 2D nonlinear photonic lattice



K. Gallo, A. Pasquazi, S. Stivala and G. Assanto, "Parametric Solitons in Two-Dimensional Lattices of Purely Nonlinear Origin", Phys. Rev. Lett. 100, 053901 (2008).

A. Pasquazi and G. Assanto Quadratic solitons in degenerate quasi-phase-matched noncollinear geometry, Phys. Rev. A 80, 021801(R) – (2009)

# Conserved quantities for QPM

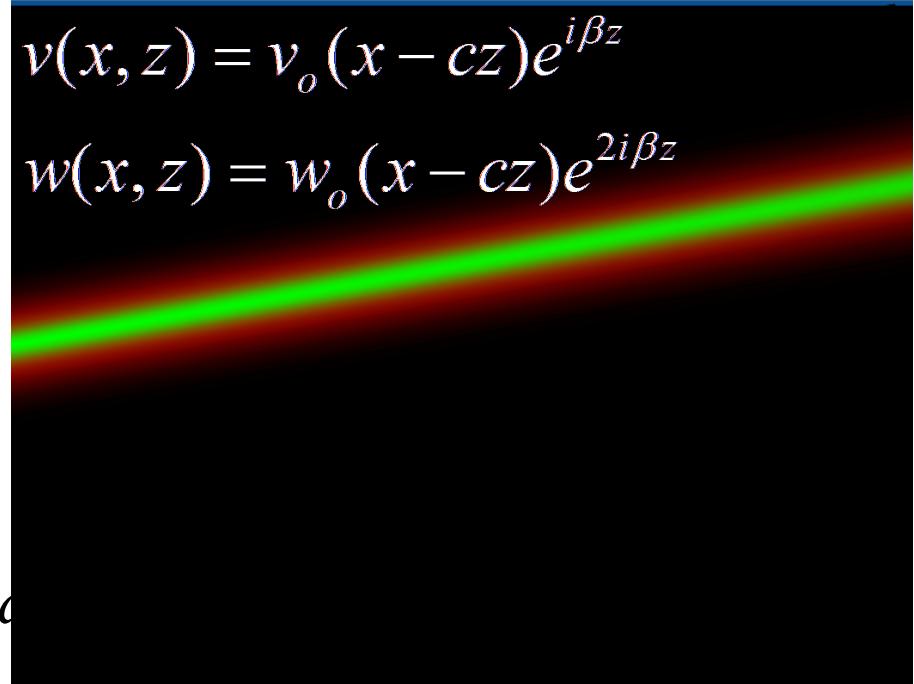
$$i\partial_z v + \frac{1}{2}\partial_{xx}v + 2v^*w = 0$$

$$i\partial_z w + \frac{1}{4}\partial_{xx}w - \Delta w + 2v^2 = 0$$

Energy 
$$Q = \int (|v|^2 + |w|^2) dx$$

Momentum 
$$M = i \int \text{Im} \left( v v_x^* + \frac{w w_x^*}{2} \right) dx$$

Hamiltonian 
$$H = \int \left( \frac{|v_x|^2}{2} + \frac{|w_x|^2}{8} \right) dx - \frac{\Delta}{2} Q_w - 2 \int \text{Re}(v^2 w^*) dx$$



# Conserved quantities for DQPM

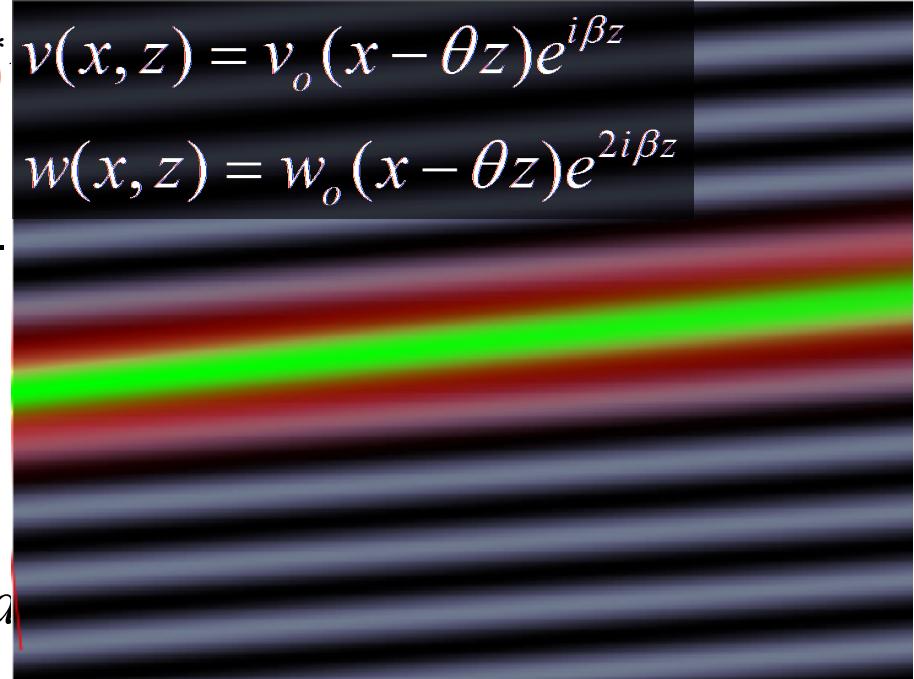
$$i\partial_z v + \frac{1}{2}\partial_{xx}v + 2\cos[\gamma(x - \theta z)]v^*$$

$$i\partial_z w + \frac{1}{4}\partial_{xx}w - \Delta w + 2\cos[\gamma(x - \theta z)]w^*$$

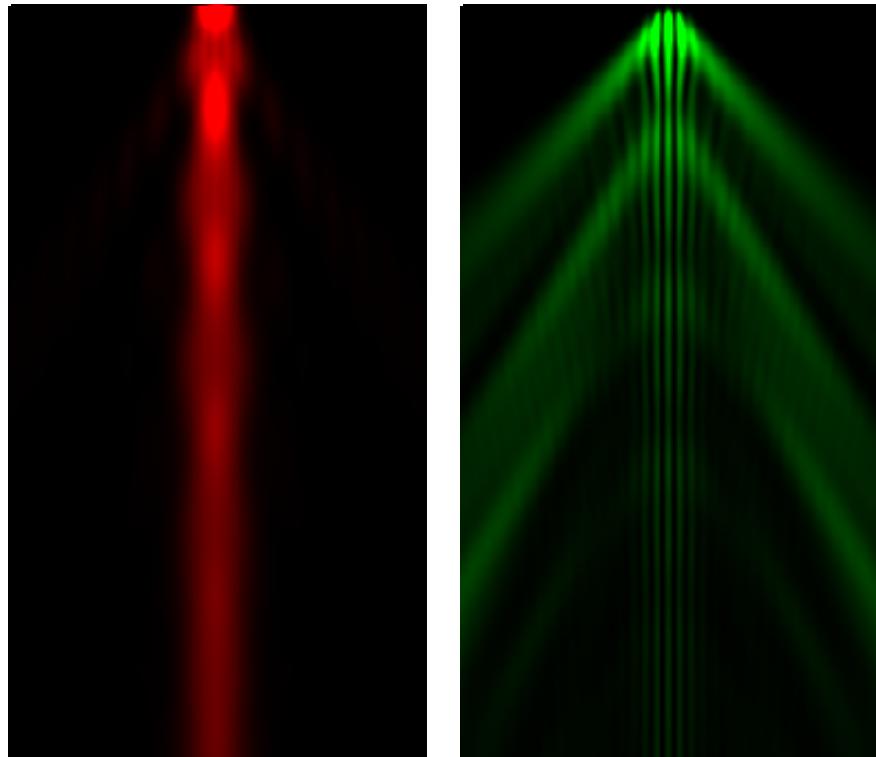
Energy 
$$Q = \int (|v|^2 + |w|^2) dx$$

Momentum 
$$M = i \int \text{Im} \left( v v_x^* + \frac{w w_x^*}{2} \right) dx$$

Hamiltonian 
$$H = \int \left( \frac{|v_x|^2}{2} + \frac{|w_x|^2}{8} \right) dx - \frac{\Delta}{2} Q_w - 2 \int \cos[\gamma(x - \theta z)] \text{Re}(v^2 w^*) dx$$



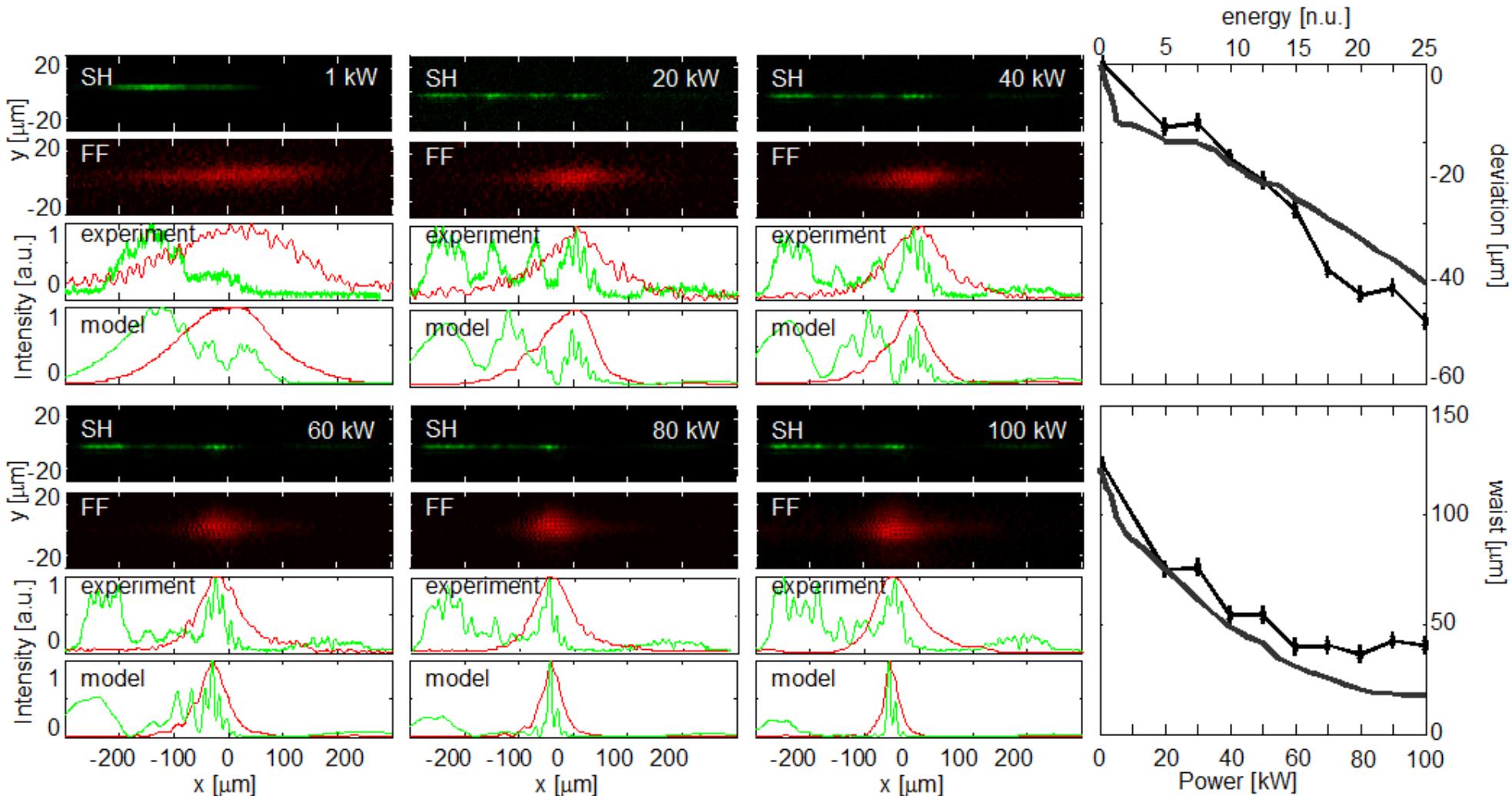
# Degenerate QPM: a new class of solitary waves



K. Gallo, A. Pasquazi, S. Stivala and G. Assanto, "Parametric Solitons in Two-Dimensional Lattices of Purely Nonlinear Origin", Phys. Rev. Lett. 100, 053901 (2008).

A. Pasquazi and G. Assanto Quadratic solitons in degenerate quasi-phase-matched noncollinear geometry, Phys. Rev. A 80, 021801(R) – (2009)

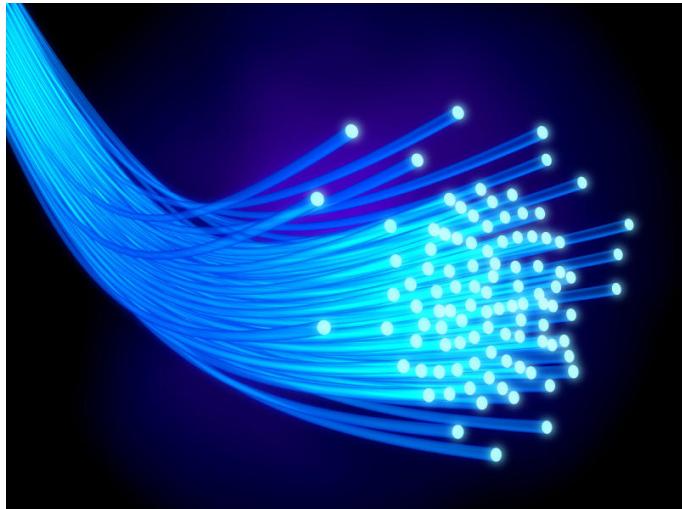
# Degenerate QPM: a new class of solitary waves



K. Gallo, A. Pasquazi, S. Stivala and G. Assanto, "Parametric Solitons in Two-Dimensional Lattices of Purely Nonlinear Origin", Phys. Rev. Lett. 100, 053901 (2008).

A. Pasquazi and G. Assanto Quadratic solitons in degenerate quasi-phase-matched noncollinear geometry, Phys. Rev. A 80, 021801(R) – (2009)

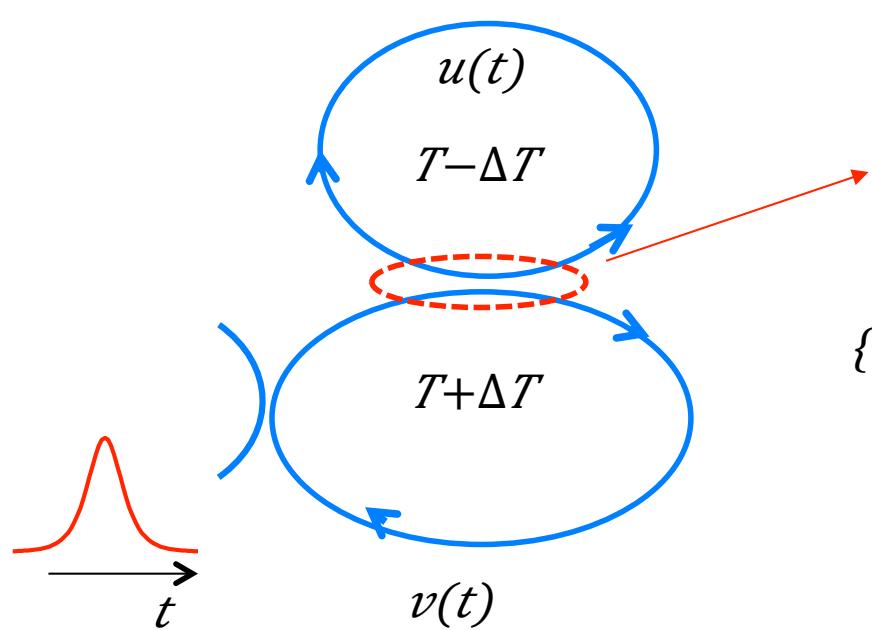
## *.. Recipe n 2 to build a D(N)LSE*



- 1) Take 2 waveguides
- 2) Put them close
- 3) Close them in a loop
- 4) Watch the propagation of a pulse

$$u \downarrow n \uparrow m+1 - u \downarrow n \uparrow m-1 = i\sqrt{2} [u \downarrow n+1 \uparrow m + u \downarrow n-1 \uparrow m ]$$
$$2 \partial \downarrow m u \downarrow n = i\sqrt{2} [u \downarrow n+1 + u \downarrow n-1 ]$$

# Synthetic Photonic Lattices



$$\{ \blacksquare u_{\downarrow} \text{out}(t) = 1/\sqrt{2} [i u_{\downarrow} \text{in}(t) + v_{\downarrow} \text{in}(t)] @ v_{\downarrow} \text{out}$$

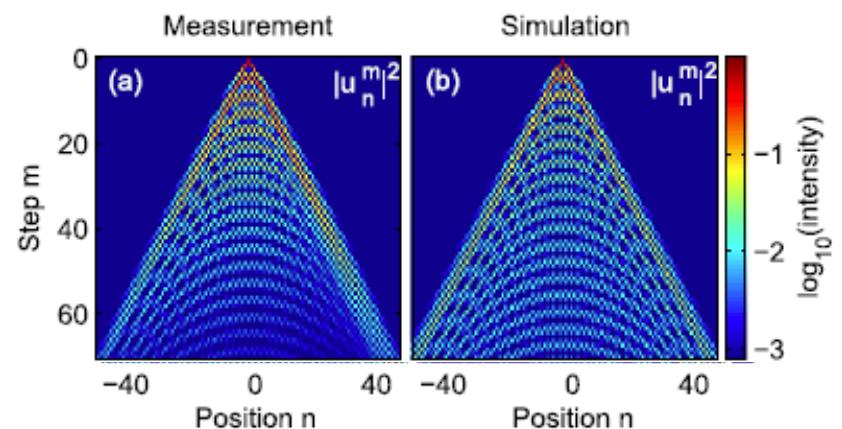
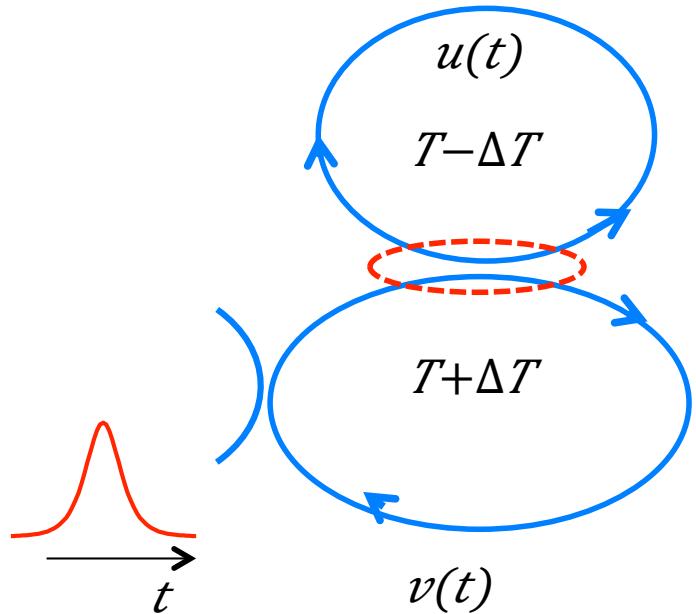
$$\{ \blacksquare u(t) = 1/\sqrt{2} [i u(t-T+\Delta T) + v(t-T-\Delta T)] @ v(t) =$$

$$v(t) = \sum m \uparrow \sum n=1 \uparrow N v_{\downarrow} n \uparrow m (t - mT + n \Delta T - 2\Delta T) \quad u(t) = \sum m \uparrow \sum n=1 \uparrow N u_{\downarrow} n \uparrow m (t - mT + n \Delta T - 2\Delta T)$$

$$\{ \blacksquare u_{\downarrow} n \uparrow m+1 (t) = 1/\sqrt{2} [i u_{\downarrow} n+1 \uparrow m + v_{\downarrow} n+1 \uparrow m] @ v_{\downarrow} n \uparrow m+1 (t) = 1/\sqrt{2} [u_{\downarrow} n-1 \uparrow m + i v_{\downarrow} n-1 \uparrow m]$$

$$u_{\downarrow} n \uparrow m+1 - u_{\downarrow} n \uparrow m-1 = i\sqrt{2} [u_{\downarrow} n+1 \uparrow m + u_{\downarrow} n-1 \uparrow m - v_{\downarrow} n+1 \uparrow m - v_{\downarrow} n-1 \uparrow m]$$

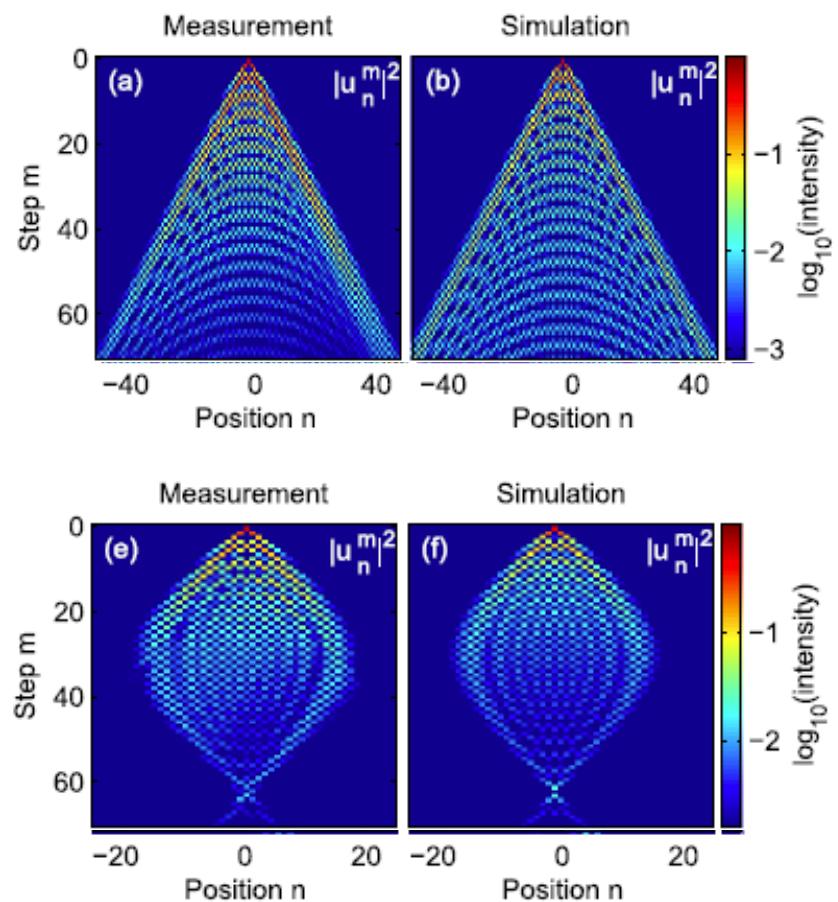
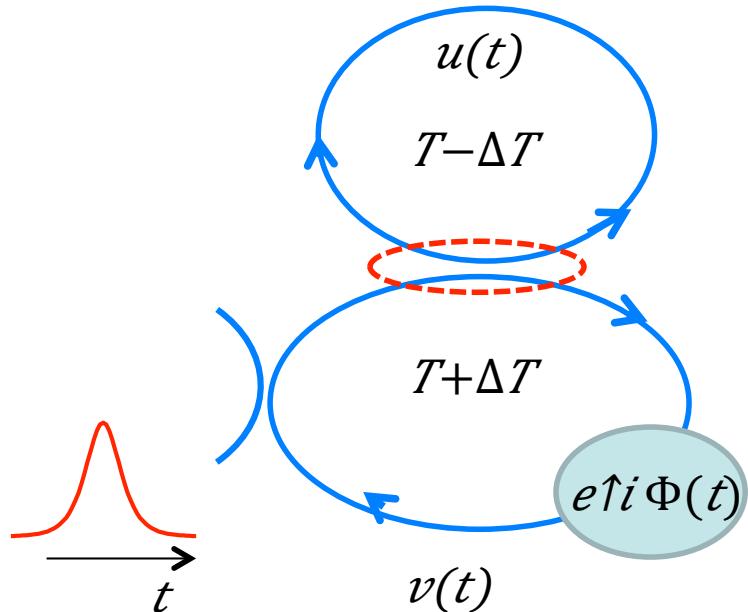
# Synthetic Photonic Lattices



Regensburger, A. et al. Photon propagation in a discrete fiber network: an interplay of coherence and losses. Phys. Rev. Lett. 107, 233902 (2011)

Regensburger, Bersch, Miri,  
Onishchukov, Christodoulides & Peschel,  
Parity-time synthetic photonic lattices,  
Nature 488, 167–171 (2012)

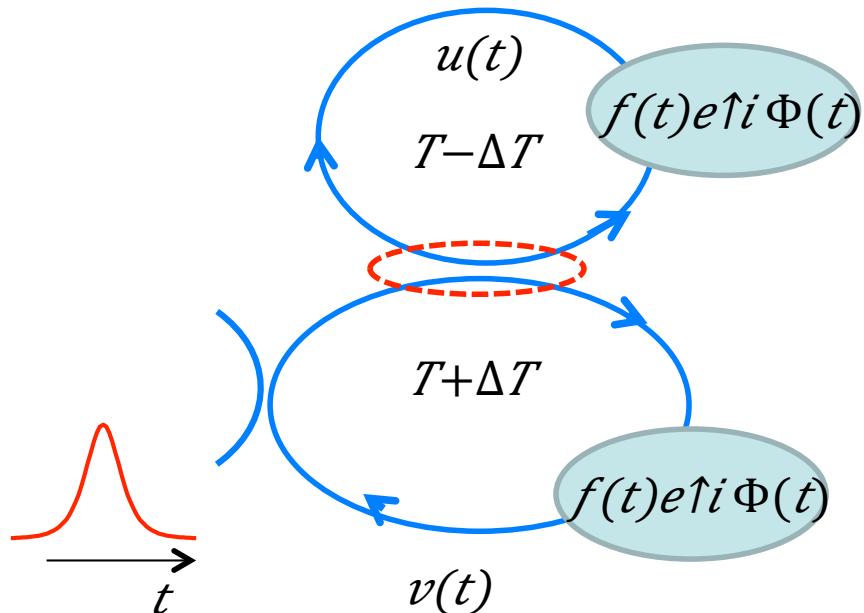
# Synthetic Photonic Lattices



Regensburger, A. et al. Photon propagation in a discrete fiber network: an interplay of coherence and losses. Phys. Rev. Lett. 107, 233902 (2011)

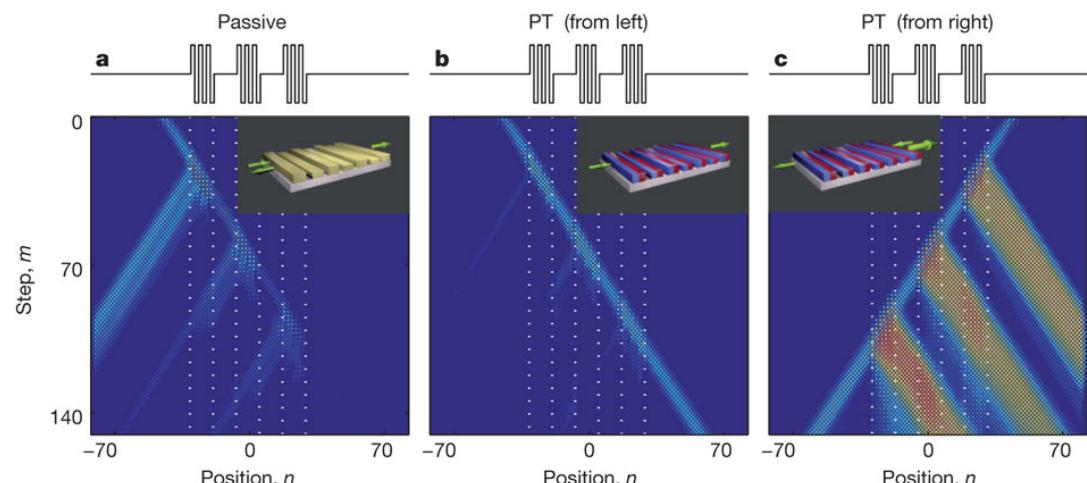
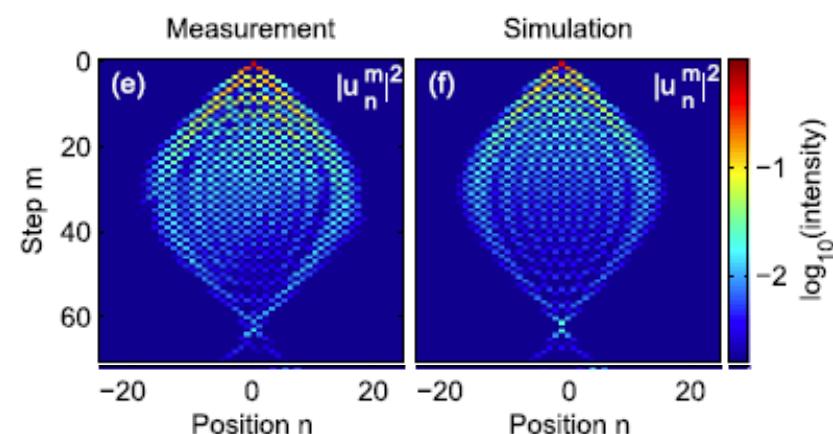
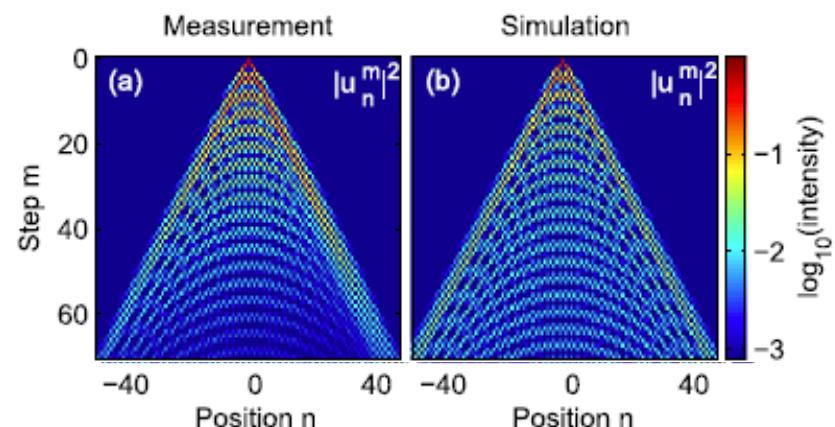
Regensburger, Bersch, Miri,  
Onishchukov, Christodoulides & Peschel,  
Parity-time synthetic photonic lattices,  
Nature 488, 167–171 (2012)

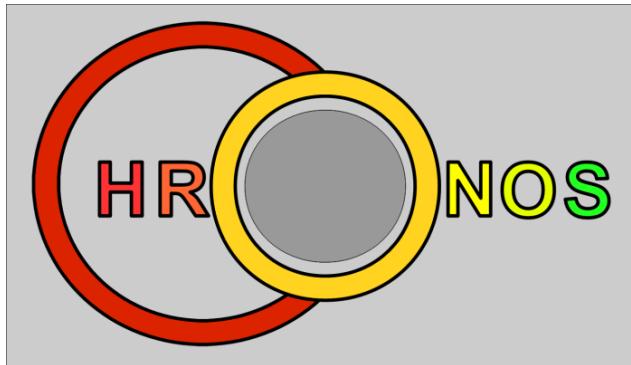
# Synthetic Photonic Lattices



Regensburger, A. et al. Photon propagation in a discrete fiber network: an interplay of coherence and losses. Phys. Rev. Lett. 107, 233902 (2011)

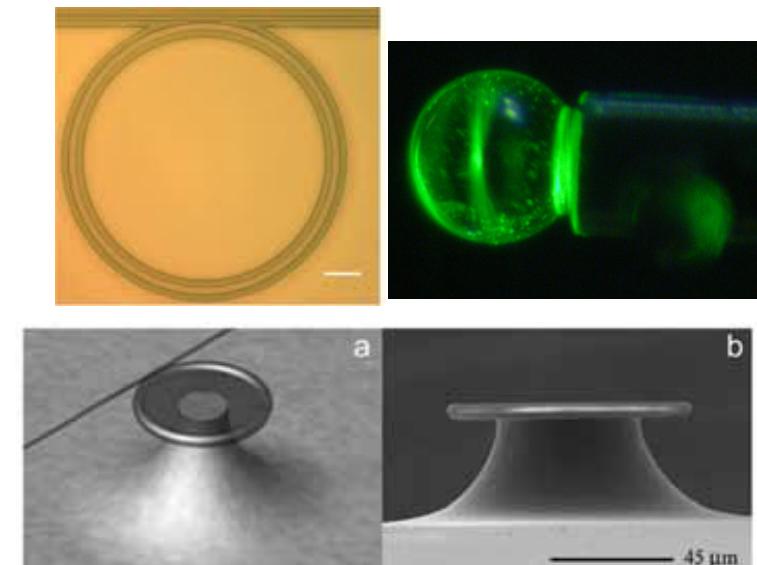
Regensburger, Bersch, Miri, Onishchukov, Christodoulides & Peschel, Parity-time synthetic photonic lattices, Nature 488, 167–171 (2012)





## On-Chip *Resonating Optical Systems via Nonlinear Optics* (CHRONOS)

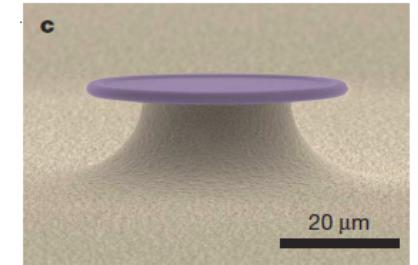
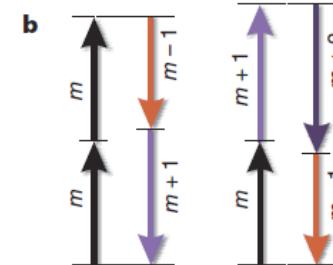
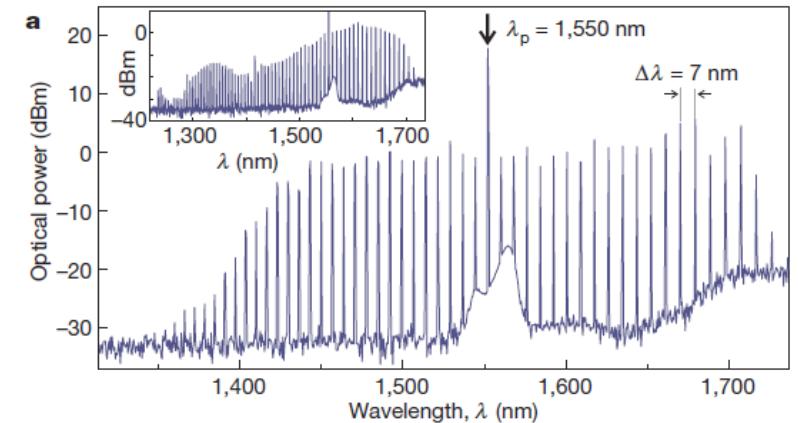
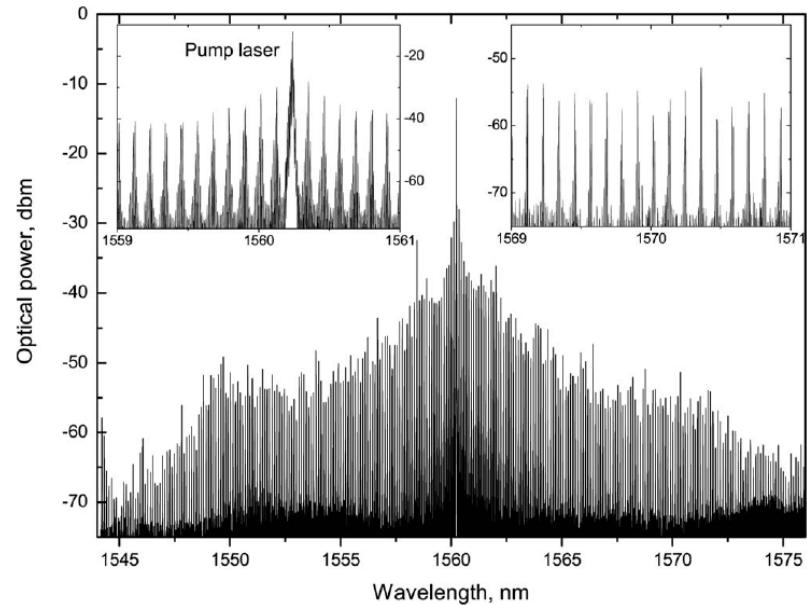
- Why Microcavities?
  - State of the art optical components
  - Already on the market (e.g. sensors)
  - Can be integrated (one of the few optical technology ready for CMOS large areas fabrication)
  - NONLINEAR WITH LOW ENERGY



# Towards Compact High Rep-Rate Sources

## OPO in High-Q resonator as Ultrafast sources

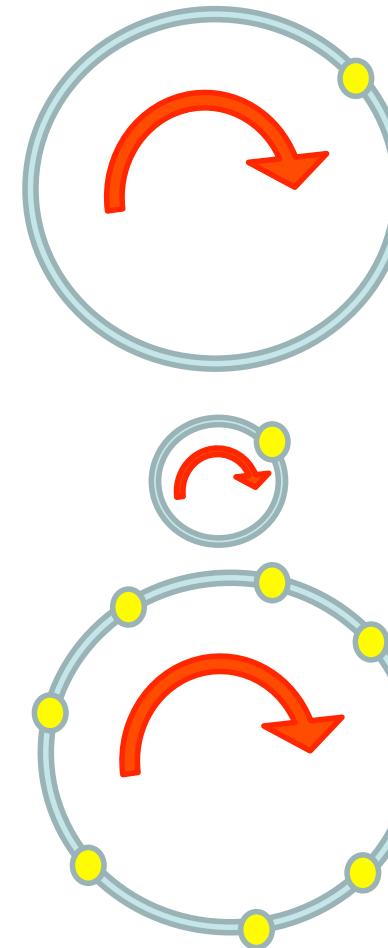
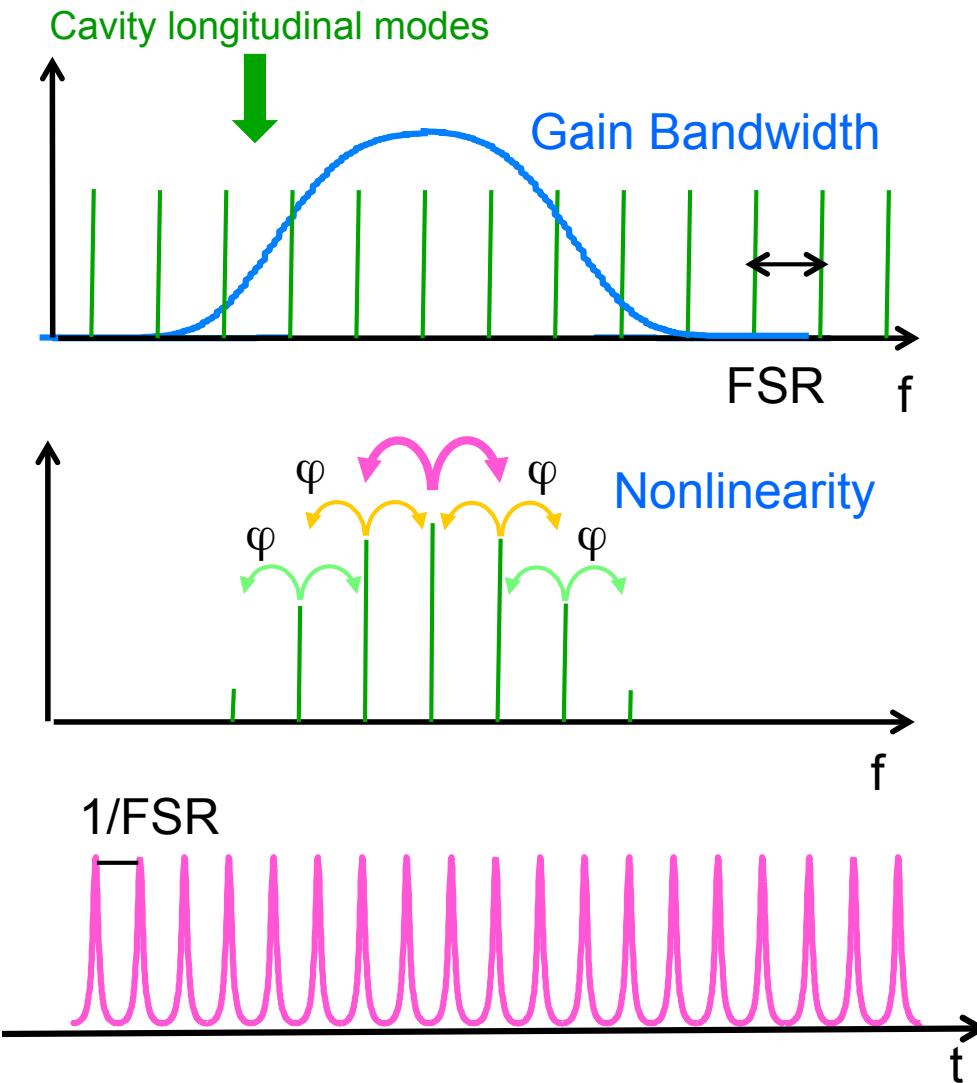
P. DelHaye et al. Nature 450, 1214–1217 (2007). FSR=870GHz



I. S. Grudinin, et al. Opt. Lett. 34, 878–880 (2009).



# Laser and Mode-Locking



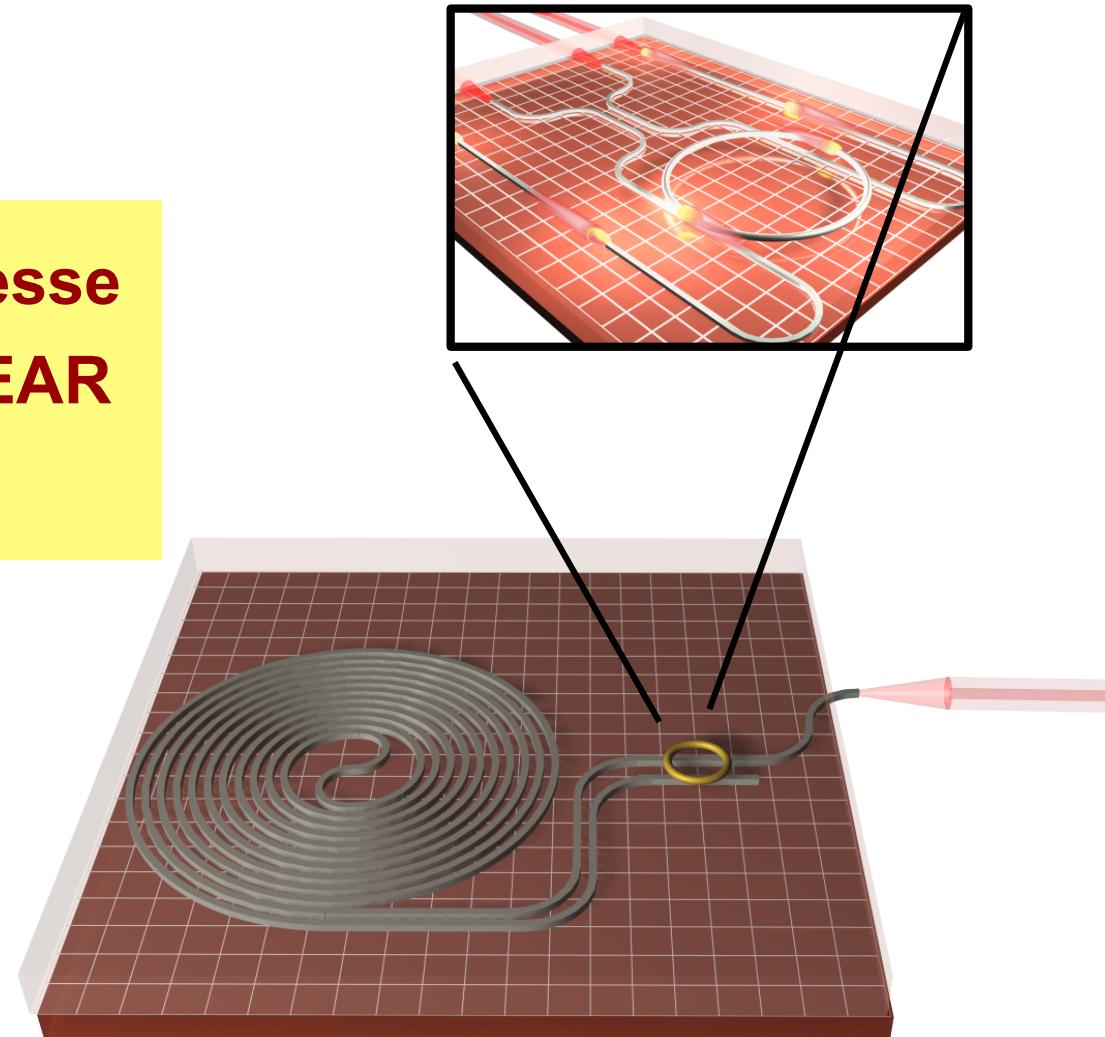
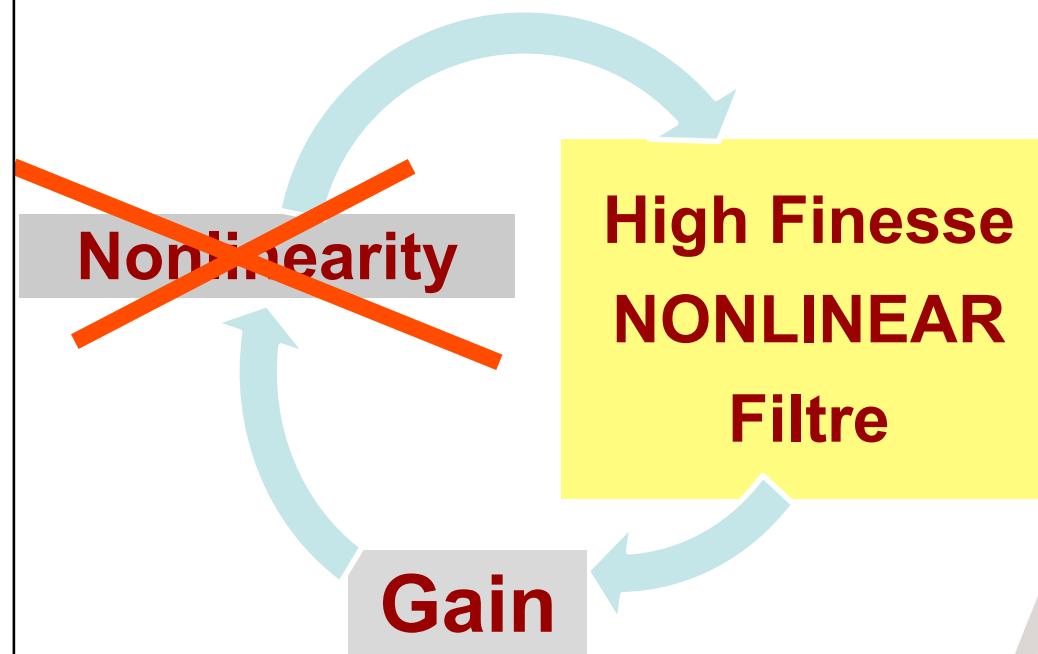
- Rep Rate
- + Gain
- + Low Noise
  
- + Rep Rate
- Gain
- High Noise
  
- + Rep Rate
- + Gain
- + Low Noise
  
- ... stability...

P. Grelu and N. Akhmediev, "Dissipative solitons for mode-locked lasers," Nat. Photonics 6, 84–92 (2012).

# Demonstration of a stable ultrafast laser based on a nonlinear microcavity

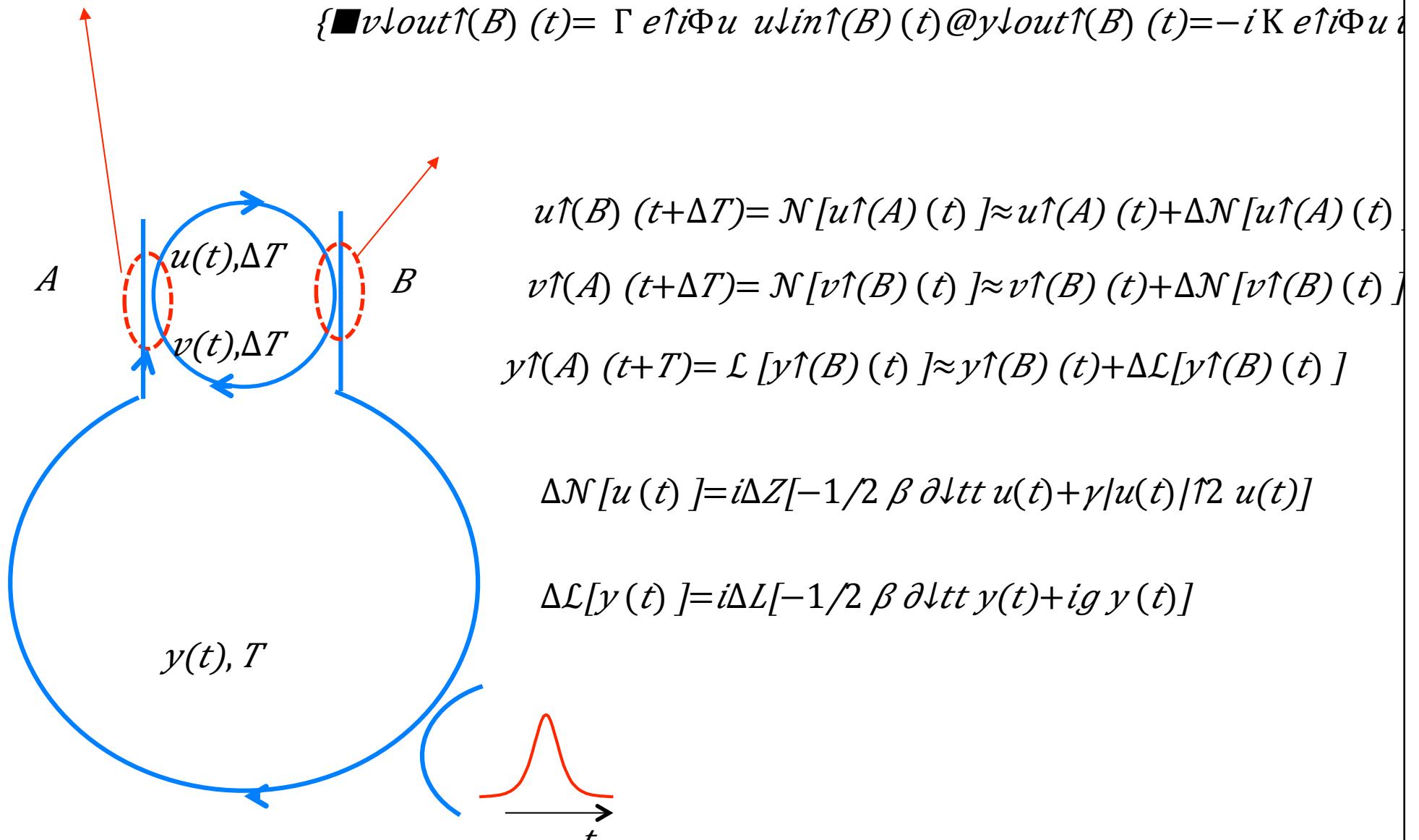
M. Peccianti<sup>1,2</sup>, A. Pasquazi<sup>1</sup>, Y. Park<sup>1</sup>, B.E. Little<sup>3</sup>, S.T. Chu<sup>3,4</sup>, D.J. Moss<sup>1,5</sup> & R. Morandotti<sup>1</sup>

*Nature Communications*  
**3, 765 (2012)**



# Synthetic Photonic Lattices

$$u \downarrow \text{out}^\uparrow(A)(t) = \Gamma e \uparrow i\Phi v v \uparrow(A) \downarrow \text{in}(t) + i K e \uparrow i\Phi y y \uparrow(A) \downarrow \text{in}(t) \Gamma \gamma_2 + K \gamma_2 = 1$$



# Synthetic Photonic Lattices

$$u \downarrow out \uparrow(A) (t) = \Gamma e \uparrow i \Phi v v \uparrow(A) \downarrow in (t) + i K e \uparrow i \Phi y y \uparrow(A) \downarrow in (t) \Gamma \gamma_2 + K \gamma_2 = 1$$

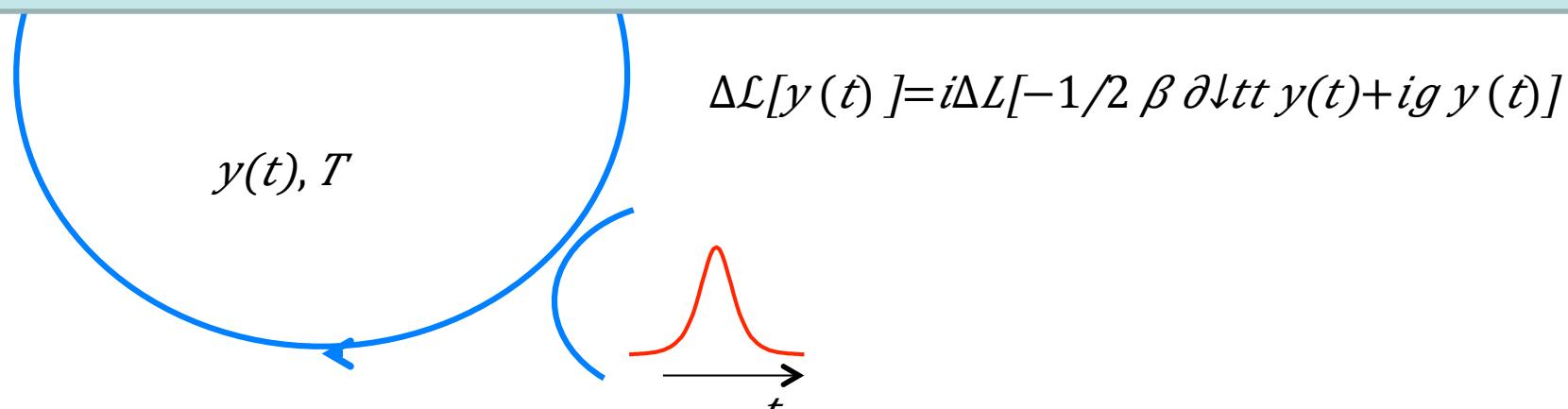


$$\{ \blacksquare v \downarrow out \uparrow(B) (t) = \Gamma e \uparrow i \Phi u u \downarrow in \uparrow(B) (t) @ y \downarrow out \uparrow(B) (t) = -i K e \uparrow i \Phi u u \downarrow in \uparrow(B) (t) \}$$

$$\mathcal{N} \uparrow - 1 [u \downarrow n+1 \uparrow m (t)] e \uparrow - i \Phi u - \Gamma e \uparrow i \Phi v \mathcal{N} [\Gamma u \downarrow n \uparrow m (t)] = K \gamma_2 \mathcal{L} [u \downarrow n \uparrow m - 1 (t)] e \uparrow i \Phi y$$

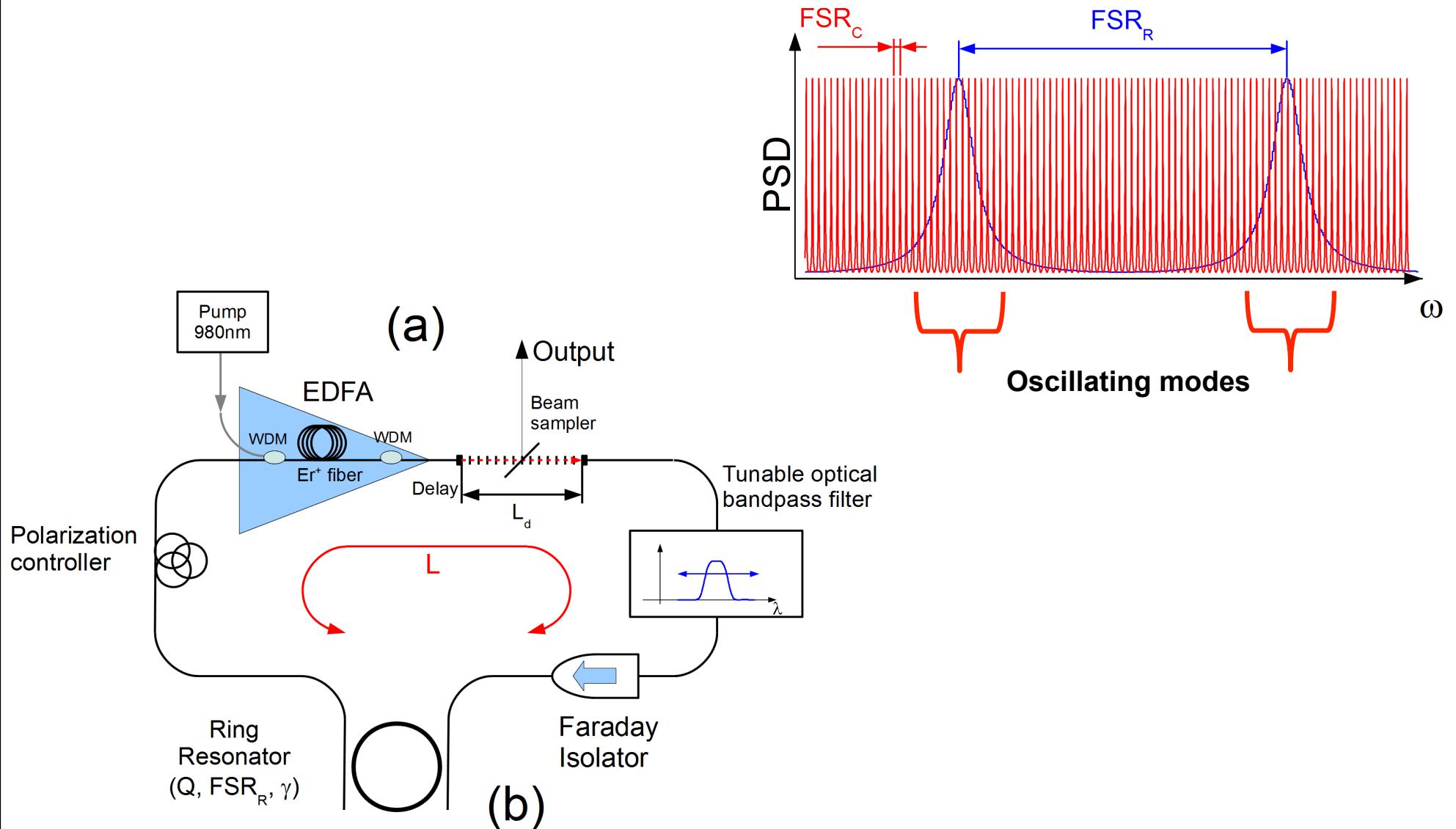
$$u \downarrow n+1 \uparrow m (t) e \uparrow - i \Phi u - \Gamma \gamma_2 u \downarrow n \uparrow m (t) e \uparrow i \Phi v - K \gamma_2 u \downarrow n \uparrow m - 1 (t) e \uparrow i \Phi y =$$

$$\Delta \mathcal{N} [u \downarrow n+1 \uparrow m (t)] e \uparrow - i \Phi u + \Gamma e \uparrow i \Phi v \Delta \mathcal{N} [\Gamma u \downarrow n \uparrow m (t)] + K \gamma_2 \Delta \mathcal{L} [u \downarrow n \uparrow m - 1 (t)] e \uparrow i \Phi y$$



# Laser setup

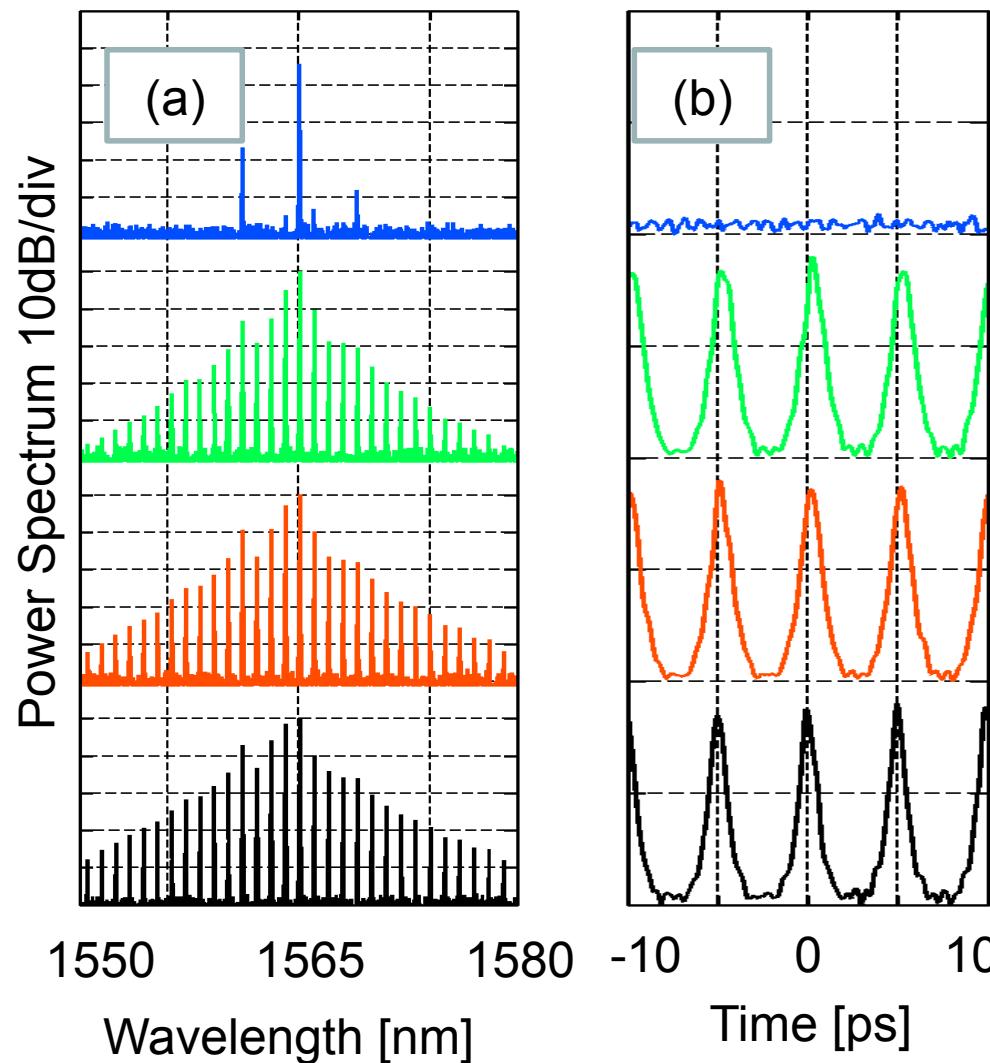
Multiple-mode per resonance oscillation



# Long main cavity: many modes per ring resonance

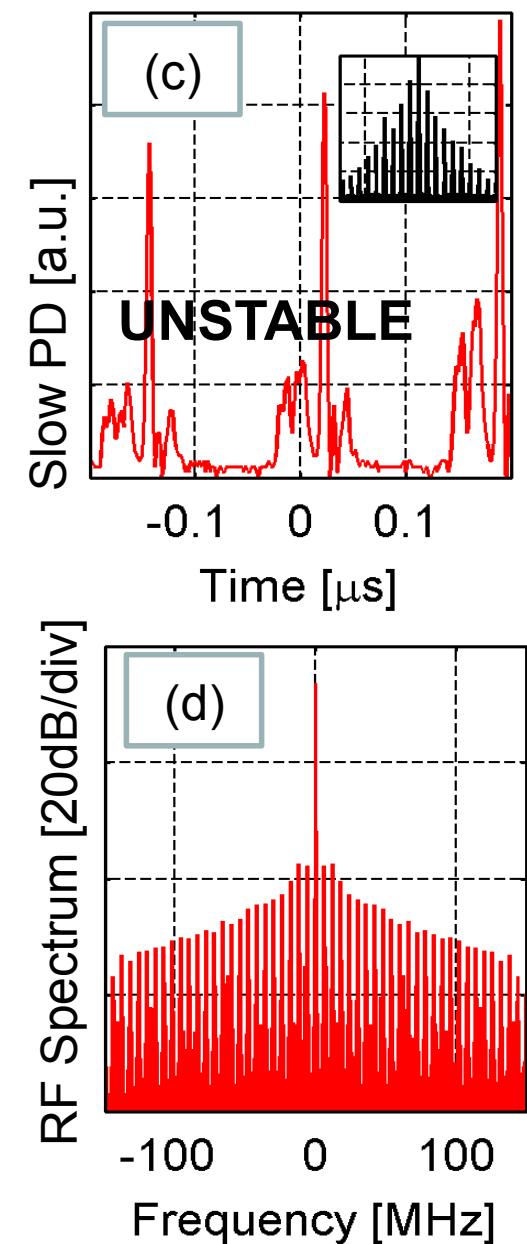
The main cavity has FSR=6MHz (33 meter of SMF).

Gain 33dB saturation power 3W



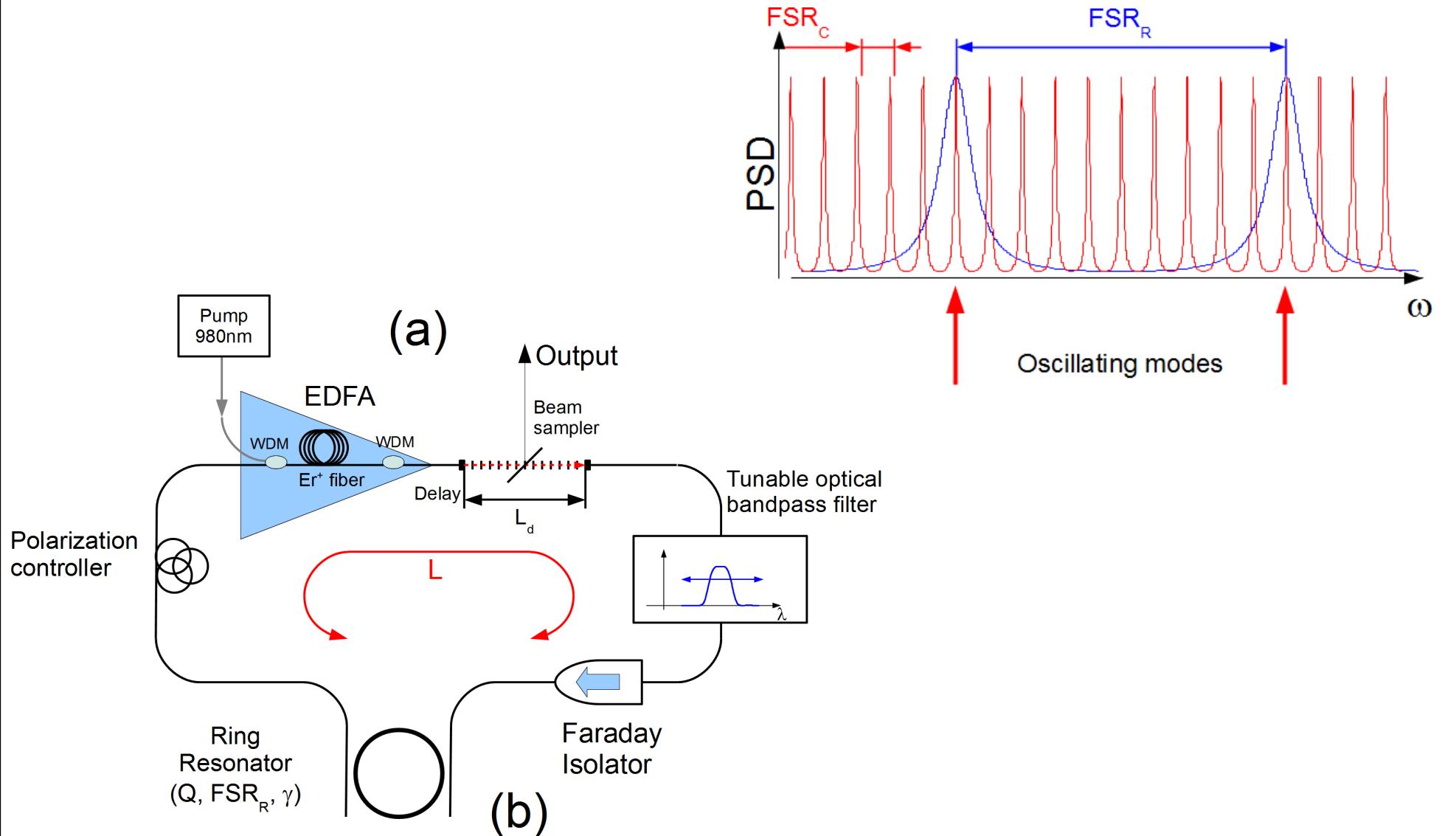
Max power in the Ring 60mW

SH -Autocorrelation [a.u.]



# Laser setup

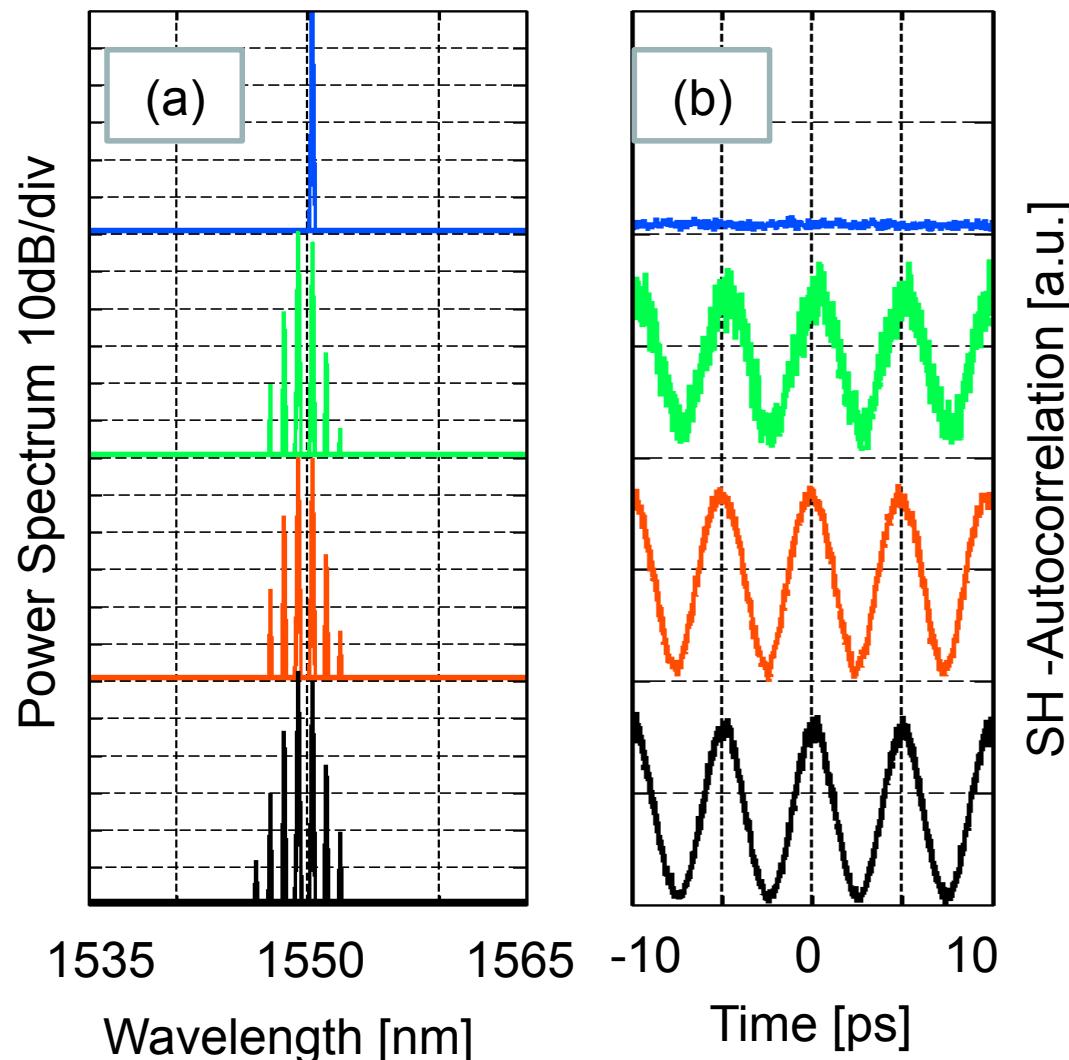
Single-mode per resonance oscillation



# Short main cavity: single-mode per ring resonance

The main cavity has FSR=67MHz (3 meter of SMF).

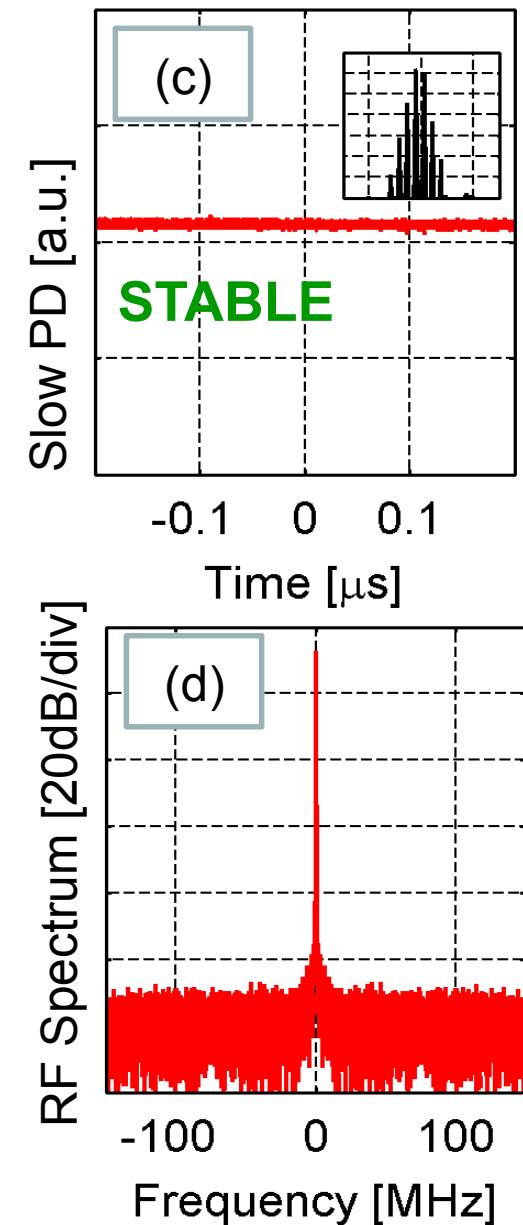
Gain 20dB saturation power 30mW

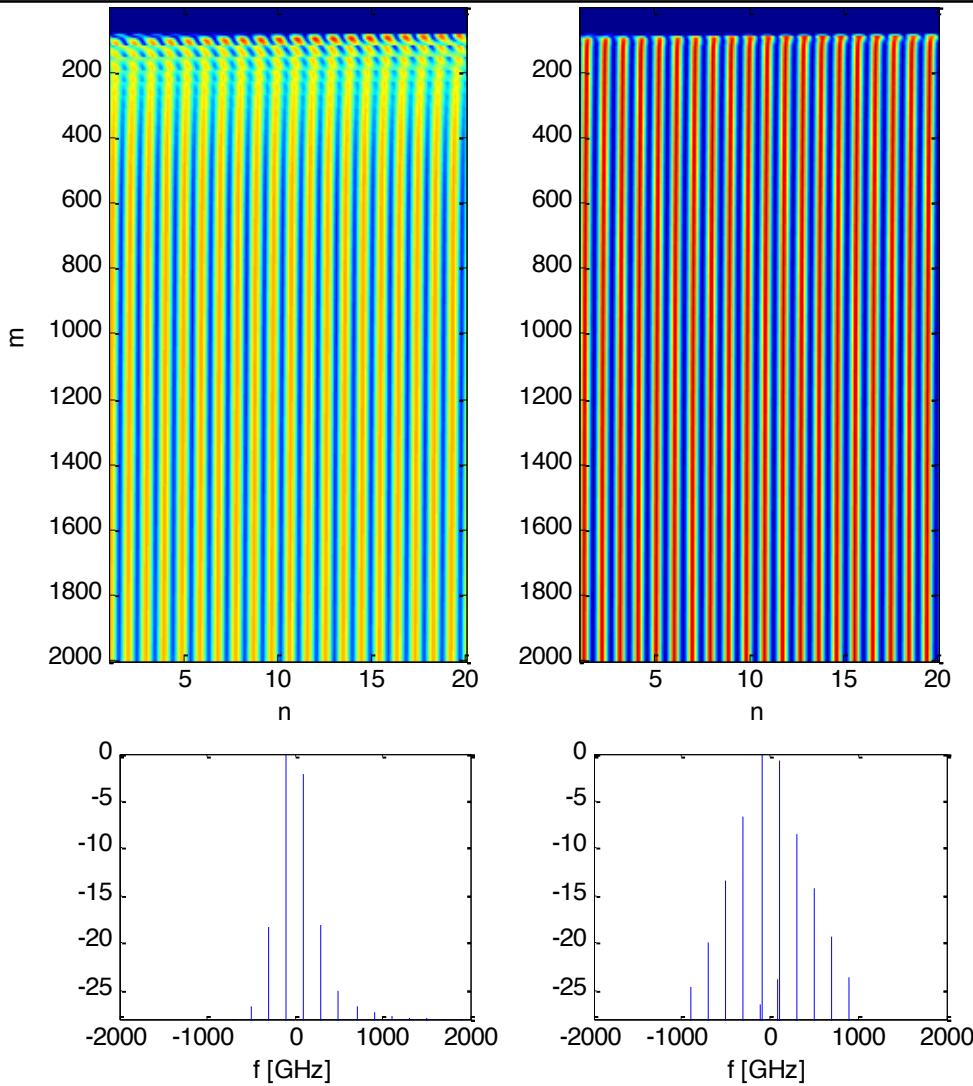


Max power in the Ring 12mW

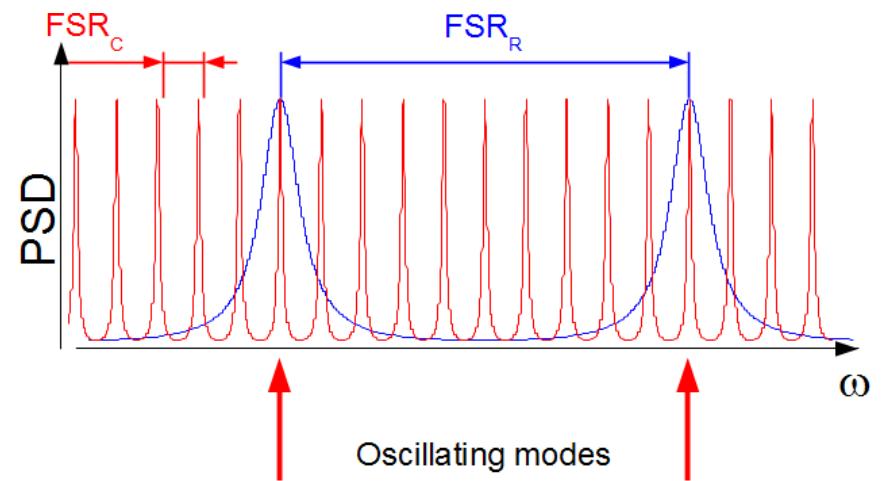
SH -Autocorrelation [a.u.]

Estimated linewidth = 13KHz !!



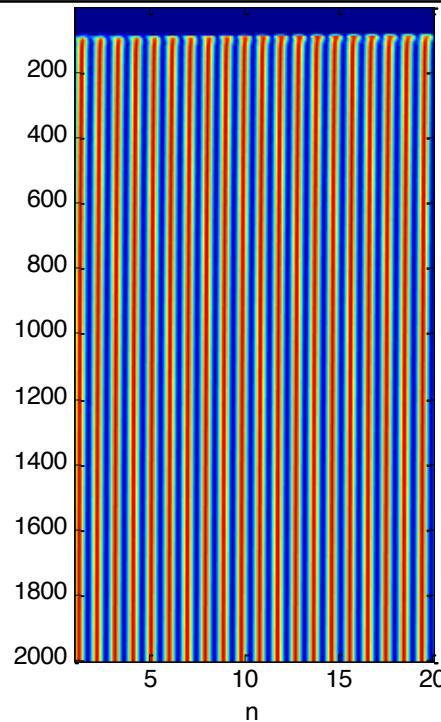
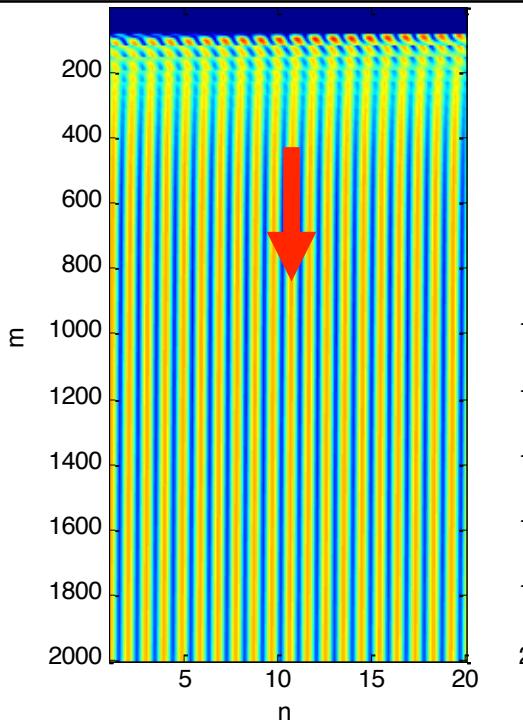


## Single-mode per resonance oscillation

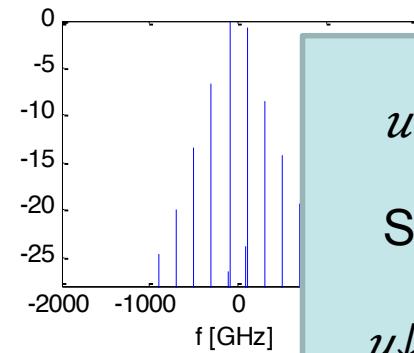
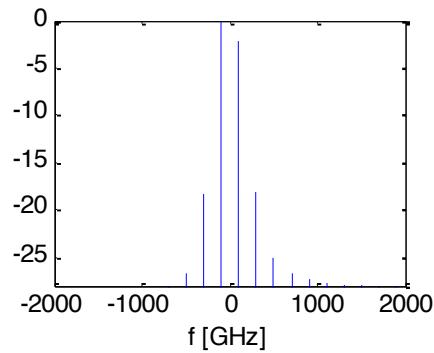
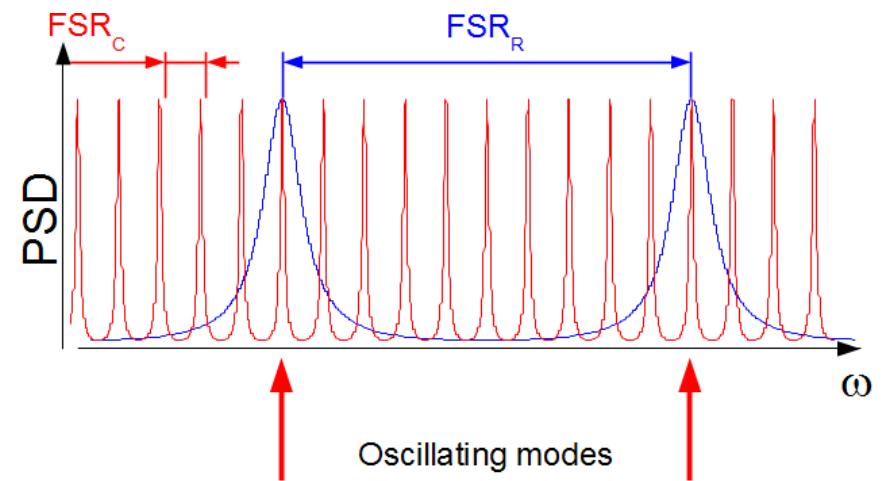


$$u \downarrow n+1 \uparrow m(t) + \Gamma \nabla^2 u \downarrow n \uparrow m(t) - K \nabla^2 u \downarrow n \uparrow m-1(t) =$$

$$\Delta \mathcal{N}[u \downarrow n+1 \uparrow m(t)] + \Gamma \Delta \mathcal{N}[\Gamma u \downarrow n \uparrow m(t)] + K \nabla^2 \Delta \mathcal{L}[u \downarrow n \uparrow m-1(t)]$$



## Single-mode per resonance oscillation



$$u \downarrow n+1 \uparrow m(t) + \Gamma \nabla^2 u \downarrow n \uparrow m(t) + K \nabla^2$$

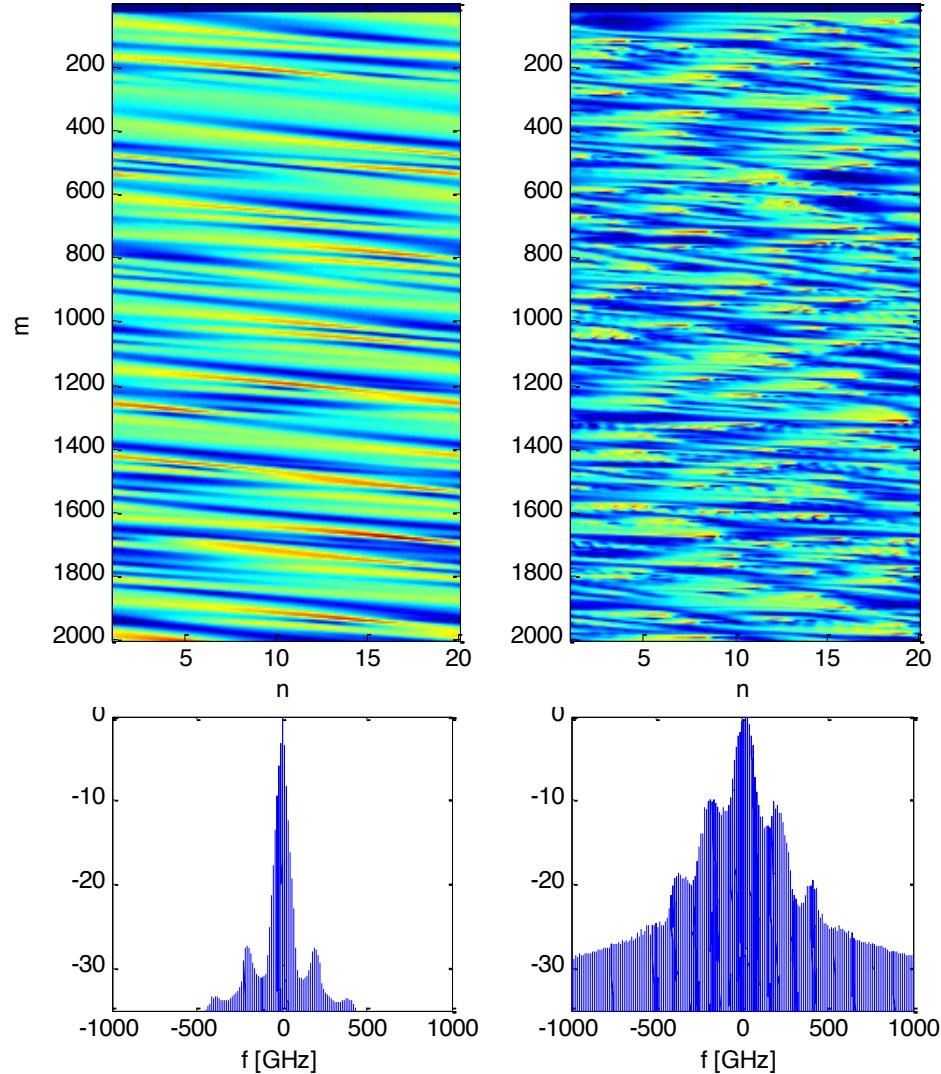
$$u \downarrow n+1 \uparrow m + \Gamma \nabla^2 u \downarrow n \uparrow m + K \nabla^2 u \downarrow n \uparrow m - 1 = 0$$

Stationary solution  $u \downarrow n = -u \downarrow n - 1 = cost$

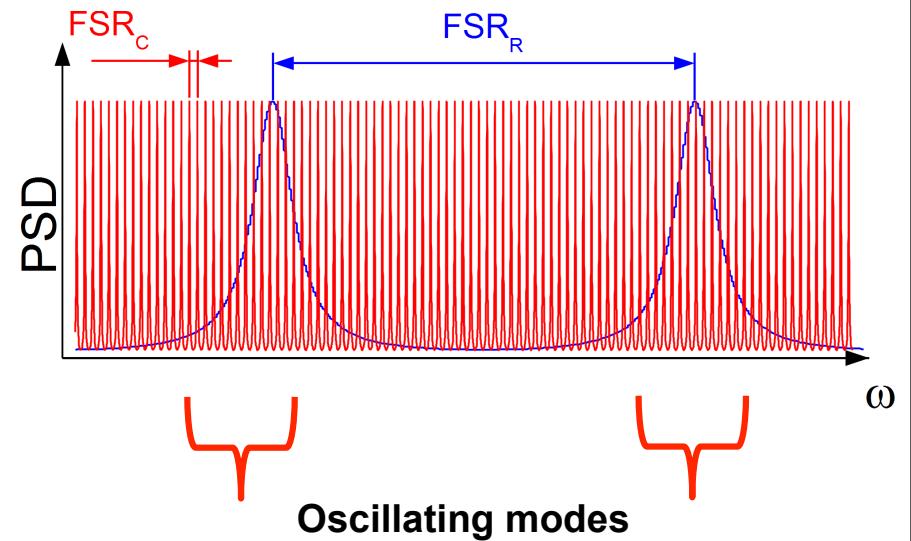
$$u \downarrow n+1 + u \downarrow n - K \nabla^2 \partial \downarrow m u \downarrow n = 0$$

Diffusive type equation

$$\Delta \mathcal{N}[u \downarrow n+1 \uparrow m(t)] - \Gamma \Delta \mathcal{N}[\Gamma u \downarrow n \uparrow m(t)] - K \nabla^2 \Delta \mathcal{L}[u \downarrow n \uparrow m - 1(t)]$$

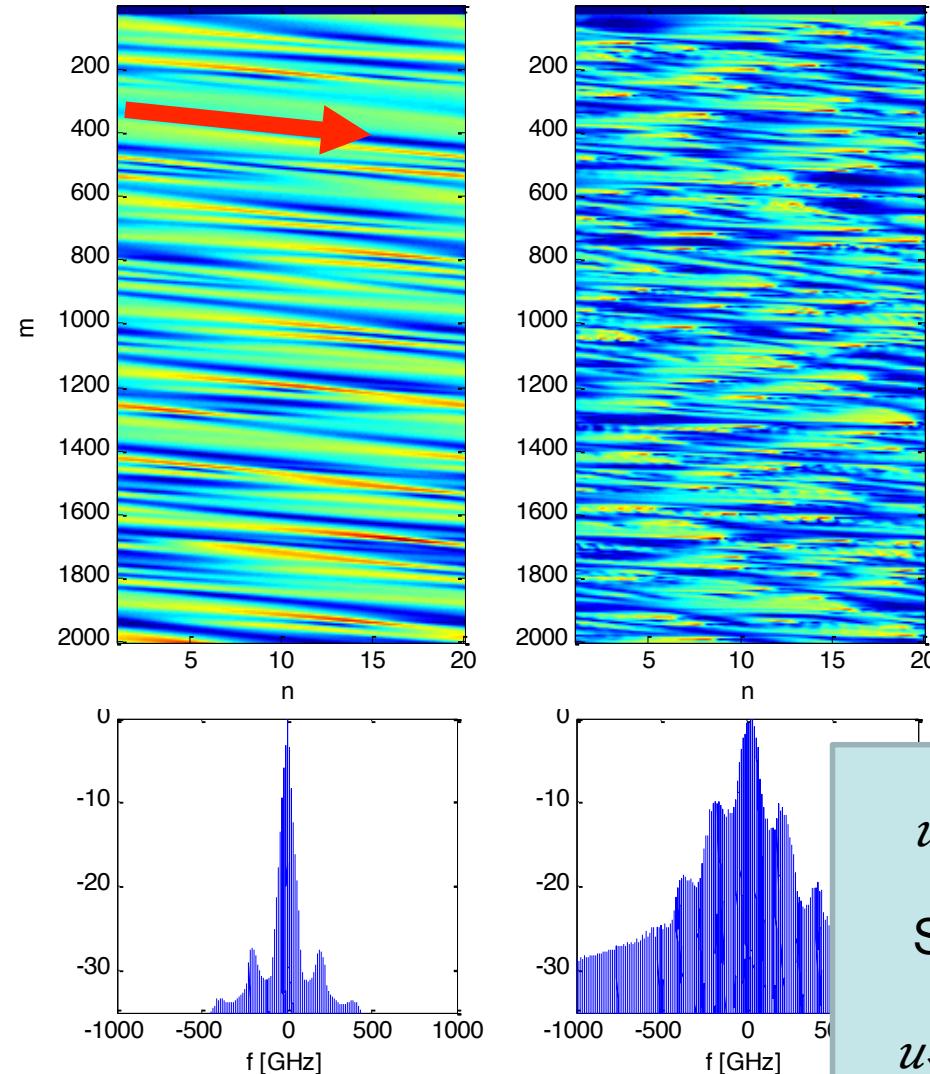


## Multiple-mode per resonance oscillation



$$u \downarrow n+1 \uparrow m(t) - \Gamma \uparrow 2 u \downarrow n \uparrow m(t) - K \uparrow 2 u \downarrow n \uparrow m-1(t) =$$

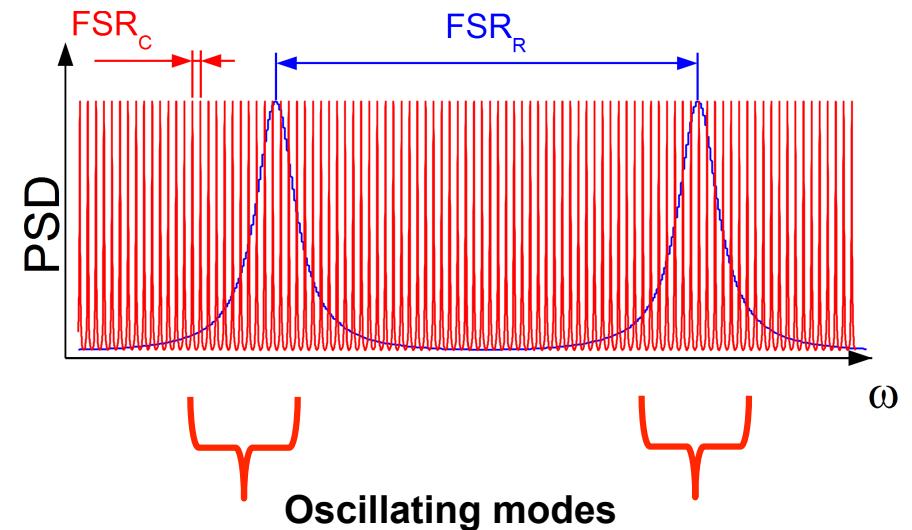
$$\Delta \mathcal{N}[u \downarrow n+1 \uparrow m(t)] + \Gamma \Delta \mathcal{N}[\Gamma u \downarrow n \uparrow m(t)] + K \uparrow 2 \Delta \mathcal{L}[u \downarrow n \uparrow m-1(t)]$$



$$u_{\downarrow n+1 \uparrow m}(t) - \Gamma \frac{1}{2} u_{\downarrow n \uparrow m}(t) - K \frac{1}{2}$$

$$\Delta \mathcal{N}[u_{\downarrow n+1 \uparrow m}(t)] + \Gamma \Delta \mathcal{N}[\Gamma u_{\downarrow n \uparrow m}(t)] + K \frac{1}{2} \Delta \mathcal{L}[u_{\downarrow n \uparrow m-1}(t)]$$

## Multiple-mode per resonance oscillation



$$u_{\downarrow n+1 \uparrow m} - \Gamma \frac{1}{2} u_{\downarrow n \uparrow m} - K \frac{1}{2} u_{\downarrow n \uparrow m-1} = 0$$

Stationary solution  $u_{\downarrow n} = \text{cost}$

$$u_{\downarrow n+1} - u_{\downarrow n} + K \frac{1}{2} \partial_{\downarrow m} u_{\downarrow n} = 0$$

Linear waves propagating with  $1/K \frac{1}{2}$  velocity

# Conclusions



- DNLS can be implemented in optical lattices
  - Purely nonlinear optical lattices have been presented and solitary waves have been studied and experimentally demonstrated

- D (N)LS can also be implemented in synthetic temporal lattices
  - Discrete modelling is an effective approach for such nested cavities design
  - Viable ways to mode lock the modes of a High-Q resonator have been demonstrated with a nested cavities approach