Negative absolute temperatures for mobile particles

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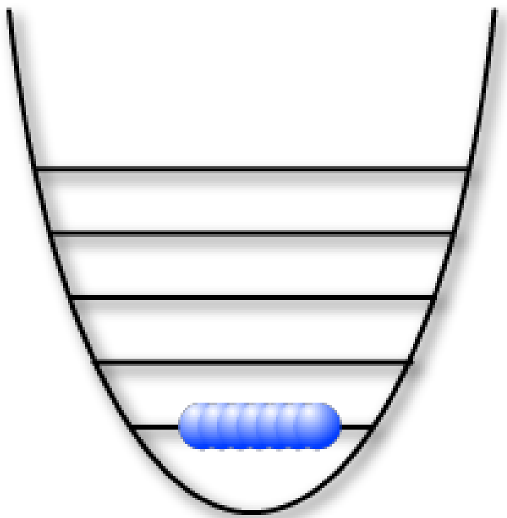
+Proposal by A. Mosk & **A. Rapp, S. Mandt and A. Rosch**

+ discussions with A. Rosch, H. Wagner, W. Zwerger, D. Huse, …

Funding: DARPA (OLE), DFG, EU, Nanosystems Initiative Munich

Our System



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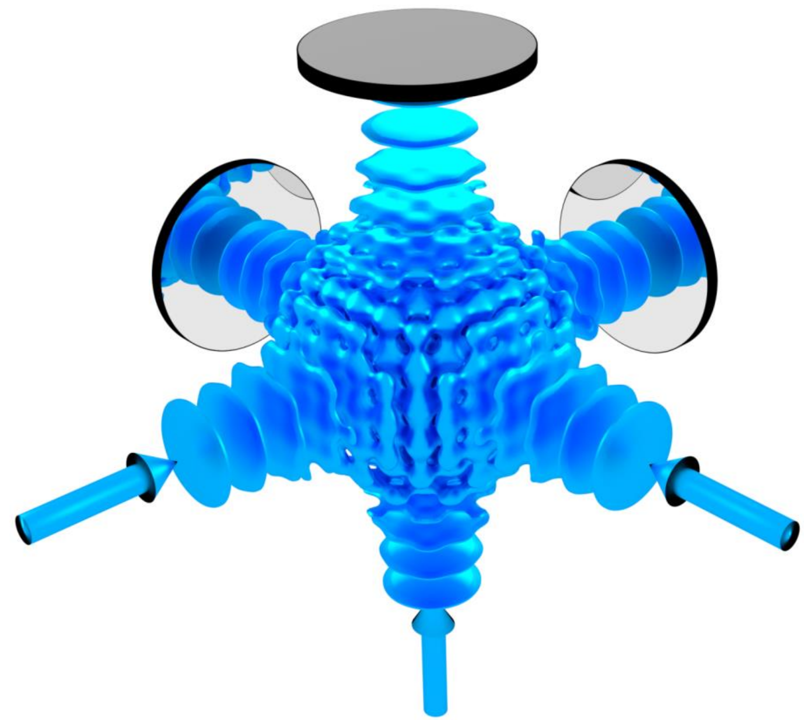
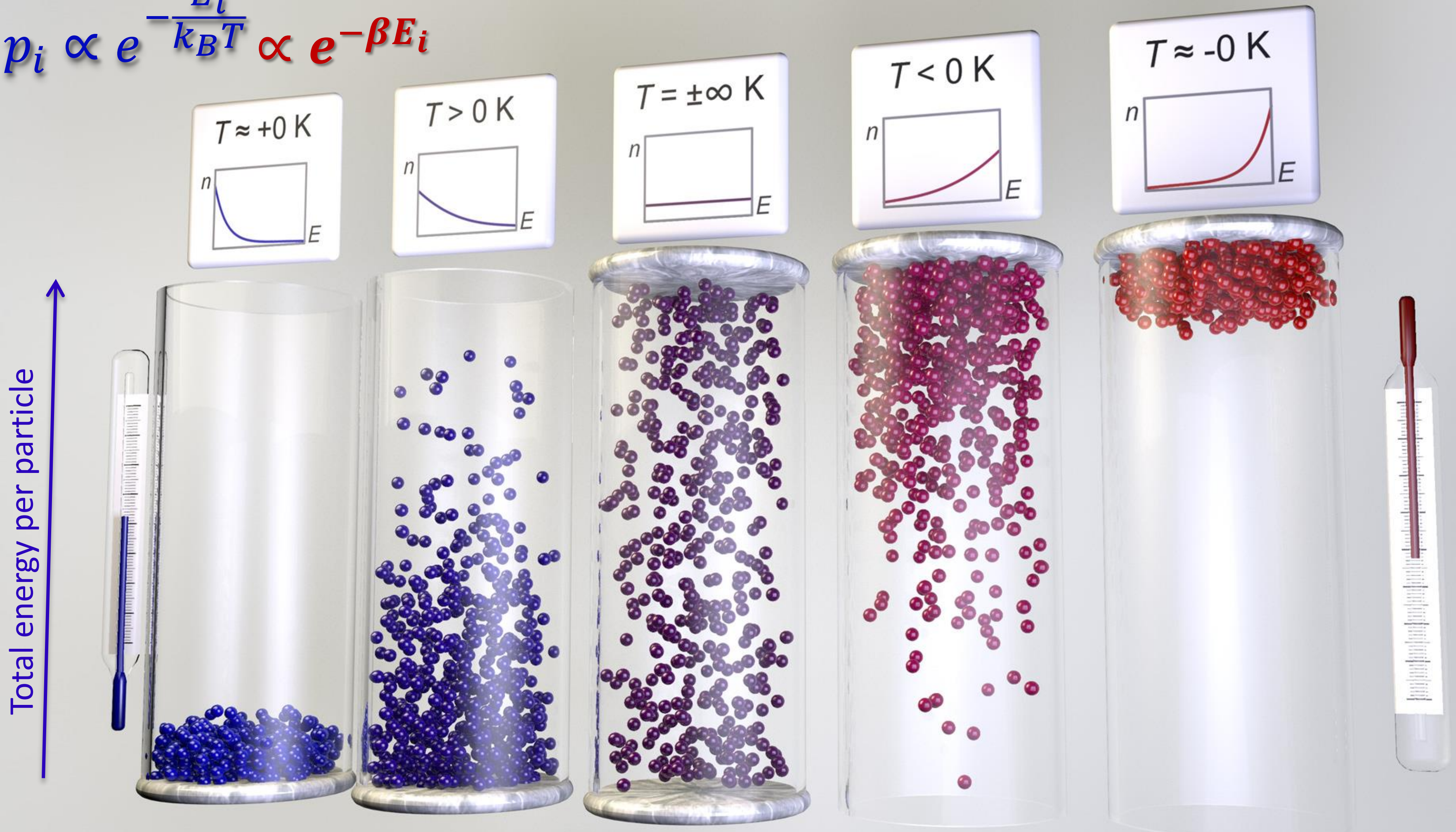


Bose-Einstein condensate
of Potassium ^{39}K atoms

3D optical lattice

Thermal states: Canonical distribution

$$p_i \propto e^{-\frac{E_i}{k_B T}} \propto e^{-\beta E_i}$$



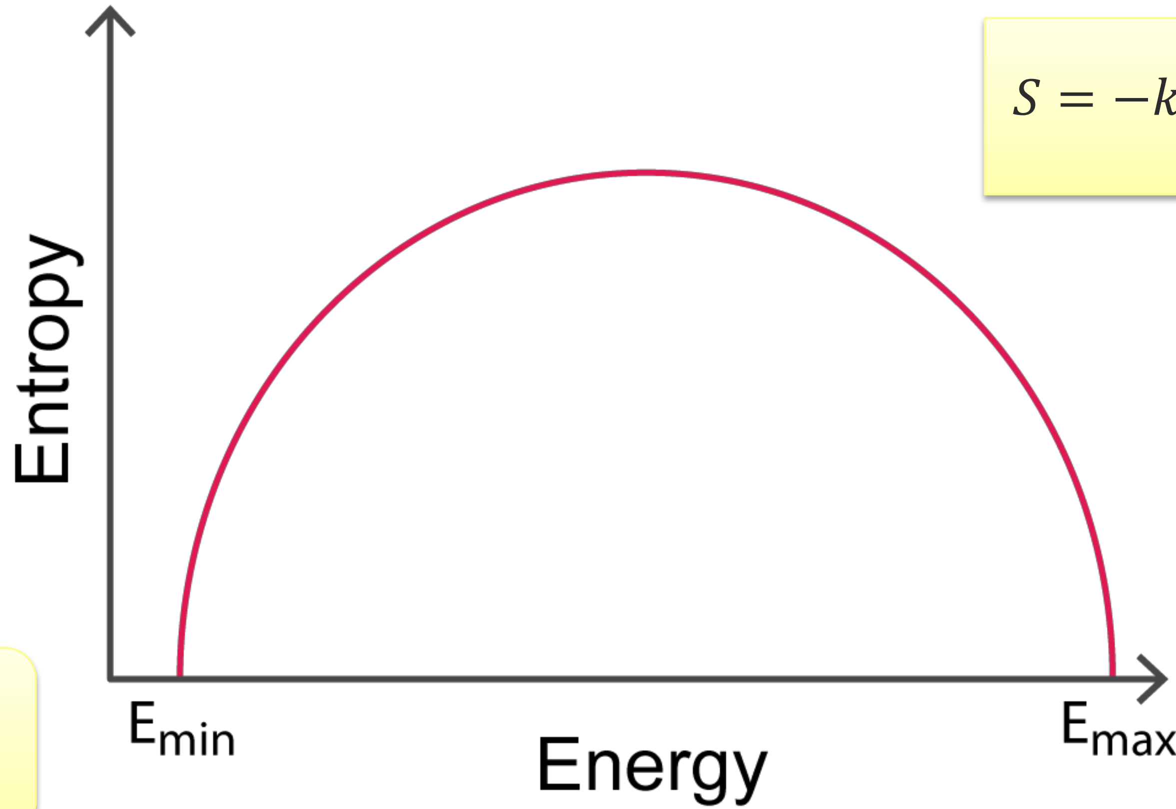
Negative Temperatures are *hotter* than all positive temperatures

(continuous + monotonously increasing scale: $-\beta = -\frac{1}{k_B T}$)

Energy-Entropy relation

$$S = -k_B \sum_i p_i \log p_i$$

(canonical)



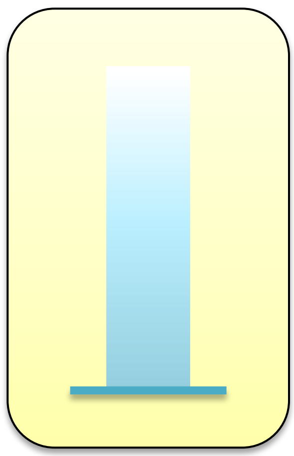
$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

$T \to 0^+$	$T \to \pm\infty$	$T \to 0^-$
$-\beta \to -\infty$	$-\beta \to 0$	$-\beta \to +\infty$

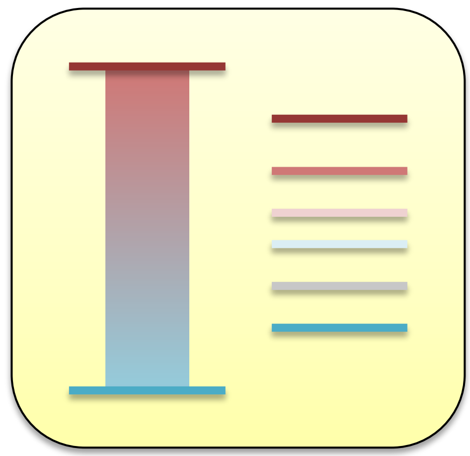
Requirement: Hamiltonian bounded from above: $\frac{E}{N} \leq \epsilon_{max}$

Applicability

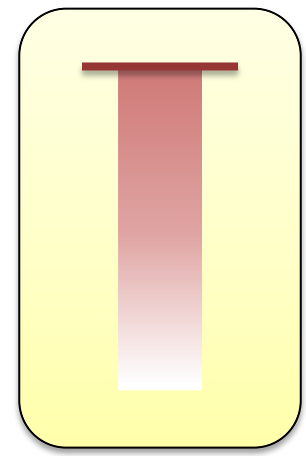
- a priori, T>0 and T<0 are equally valid



- **T>0 only**



- **T>0 and T<0**



- **T<0 only**

- Most Hamiltonians are unbounded from above, e.g. $E_{kin} \propto p^2$

  → in practice often only T>0 possible

How to get to negative Temperatures?

- Heat, Heat, Heat, ?

  Impossible: Above $T = \infty$ entropy decrease again

  → Cannot dissipate work in heat anymore

- Quasi-static state change ?

  Impossible: No (classical) adiabatic path can change sign of T (Landsberg 1959)

- **„Flip“ the energy axis:** $"\hat{H} \Rightarrow -\hat{H}"$

Previous work

A Nuclear Spin System at Negative Temperature

E. M. Purcell and R. V. Pound
Department of Physics, Harvard University, Cambridge, Massachusetts
November 1, 1950

Suddenly flipped sign of external magnetic field

PHYSICAL REVIEW VOLUME 103, NUMBER 1 JULY 1, 1956

Thermodynamics and Statistical Mechanics at Negative Absolute Temperatures

Norman F. Ramsey*
Harvard University, Cambridge, Massachusetts, and Clarendon Laboratory, Oxford, England
(Received March 26, 1956)

PRL **106,** 195301 (2011) PHYSICAL REVIEW LETTERS week ending 13 MAY 2011

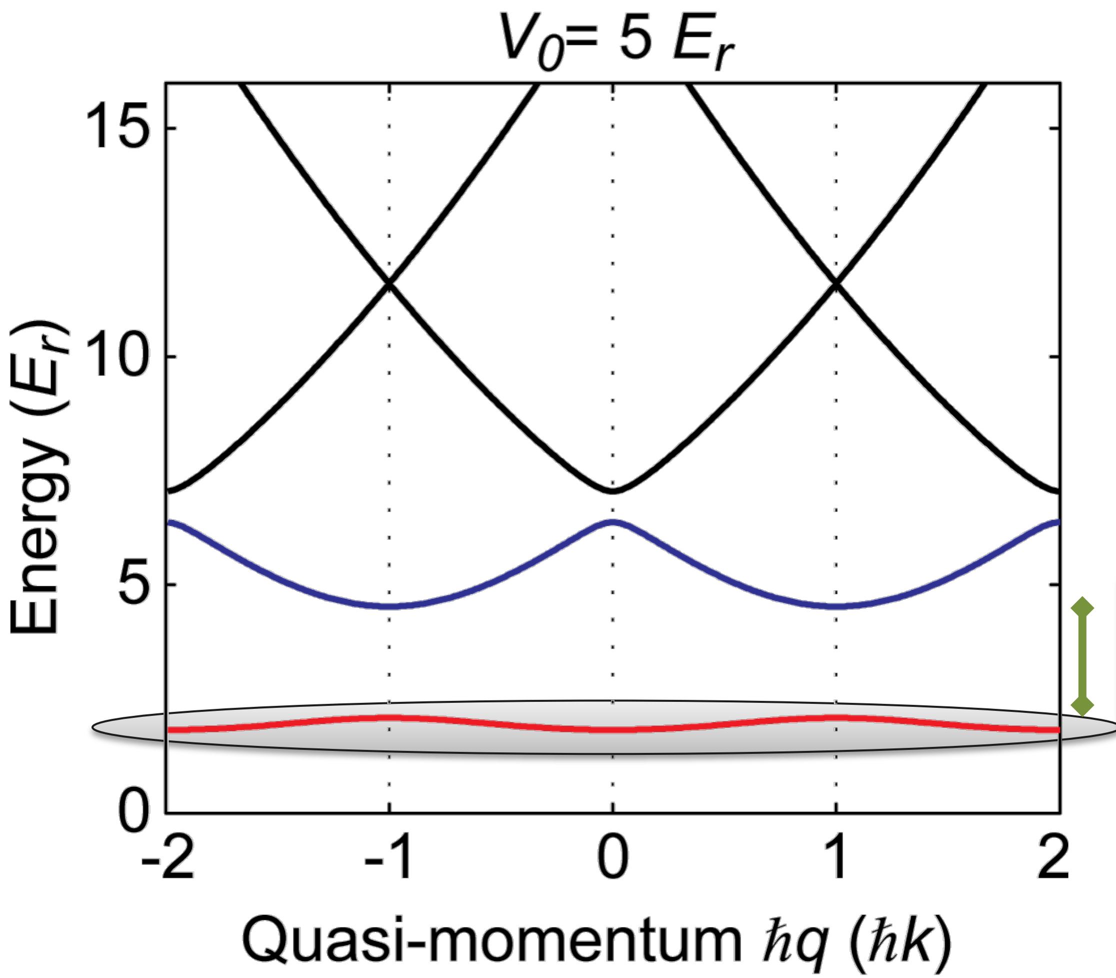
Spin Gradient Demagnetization Cooling of Ultracold Atoms

Patrick Medley,* David M. Weld,† Hirokazu Miyake, David E. Pritchard, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 12 January 2011; revised manuscript received 16 March 2011; published 12 May 2011)

We demonstrate a new cooling method in which a time-varying magnetic field gradient is applied to an ultracold spin mixture. This enables preparation of isolated spin distributions at positive and negative effective spin temperatures of ±50 pK. The spin system can also be used to cool other degrees of freedom,

- Population inversion: Basis of Laser
  → *steady state* but *no equilibrium state*

Optical lattice band structure (1D)



Band gap >> Bandwidth of lowest band

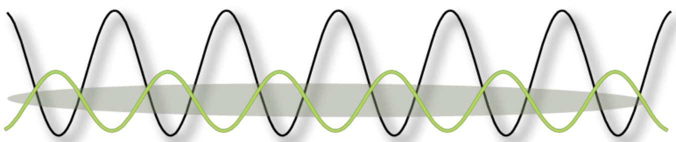
➔ kinetic energy is bounded from above and below

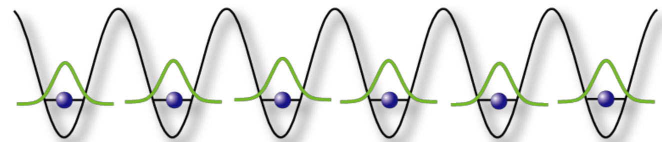
Many-body states at commensurate density

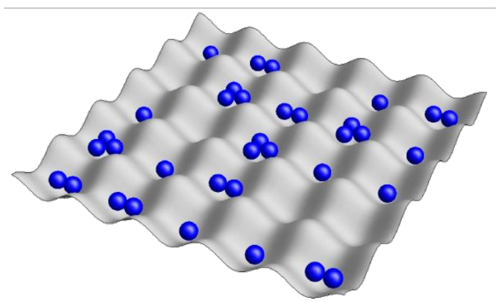
$J \gg U$: Superfluid

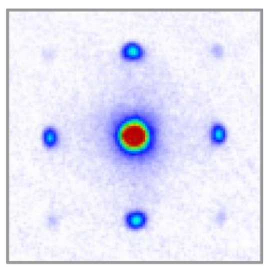
$J \ll U$: Mott Insulator

$$n = \frac{N_a}{N_l} = 1$$

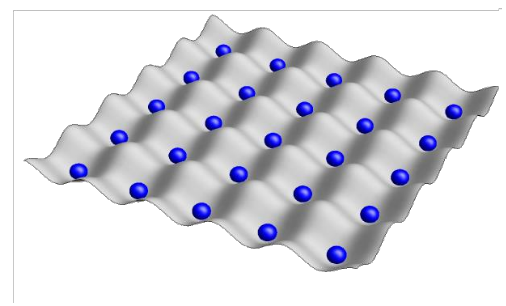


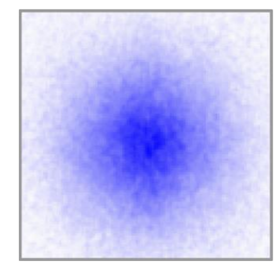






Momentum distribution

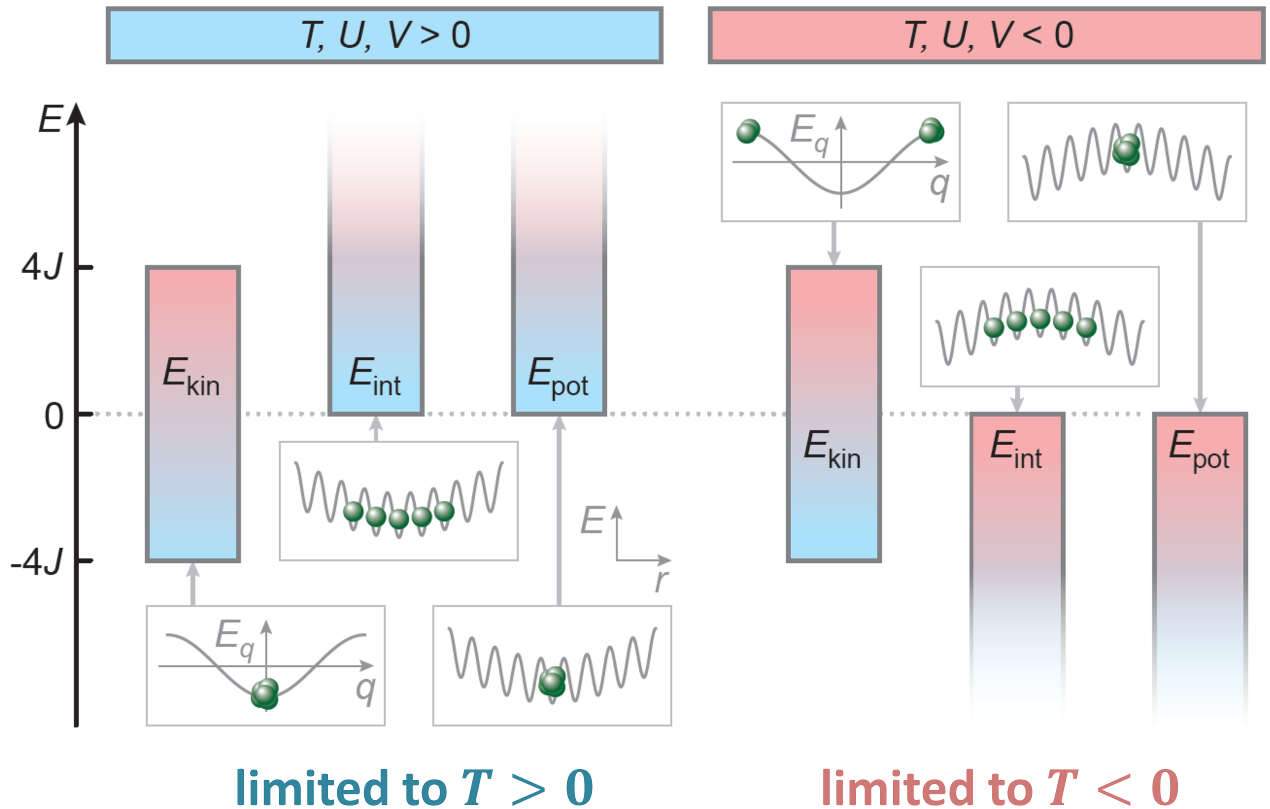




$$H = -J \sum_{\langle i,j \rangle} a_i^{+} a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

Energy bounds in Bose Hubbard Model

$$H = -\boxed{J}\sum_{\langle ij\rangle}(b_i^{+}b_j + b_j^{+}b_i) + \frac{\boxed{U}}{2}\sum_i n_i(n_i-1) + \boxed{V_0}\sum_i \mathbf{r_i}^2 n_i$$

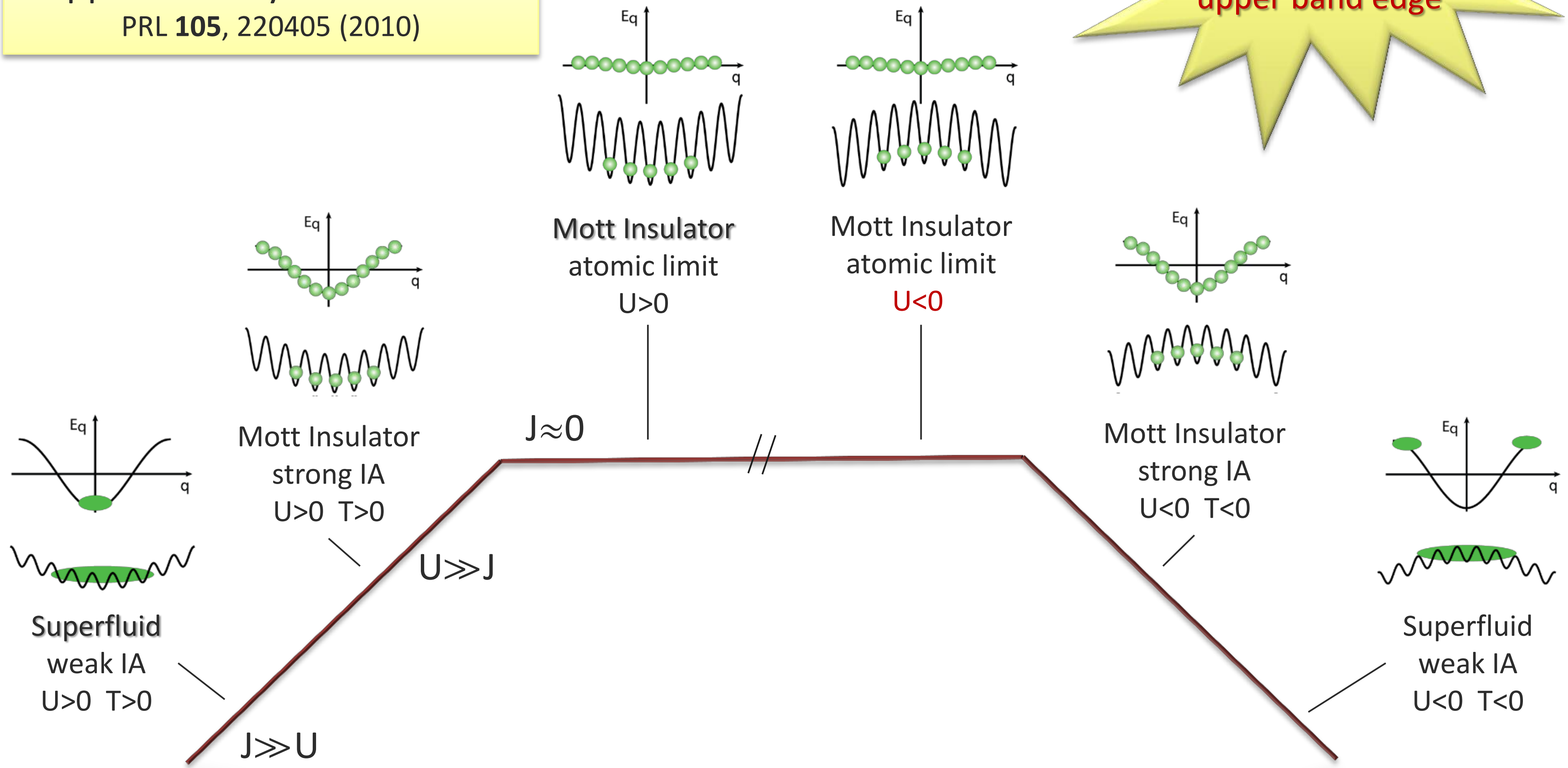


- *U* tunable via Feshbach resonance in ^{39}K
- *V* can be inverted due to *blue-detuned* optical lattice

Experimental Sequence

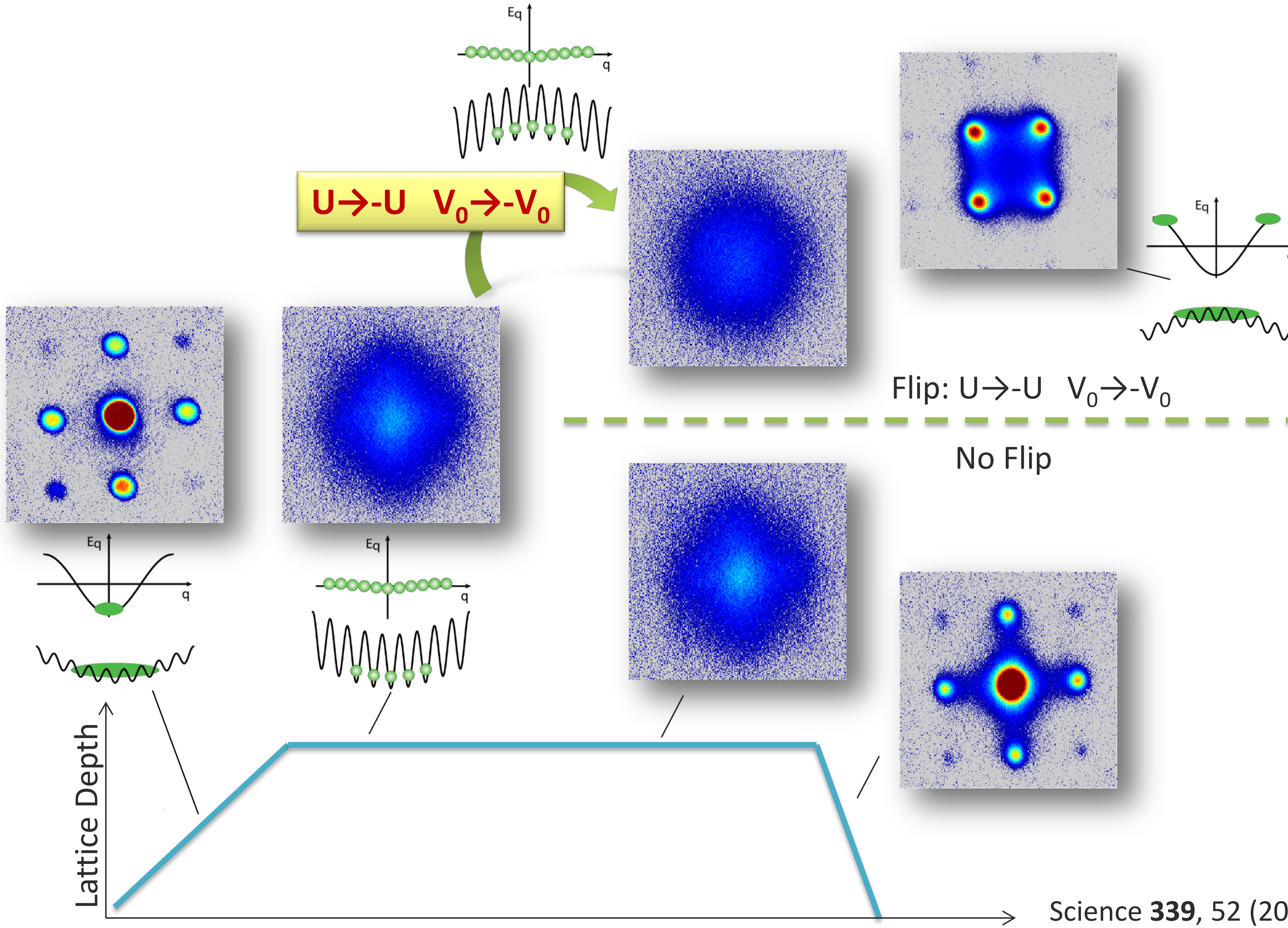
Entropy production: **Low**
Happens mainly outside of MI
PRL **105**, 220405 (2010)

Stable Bose gas at upper band edge



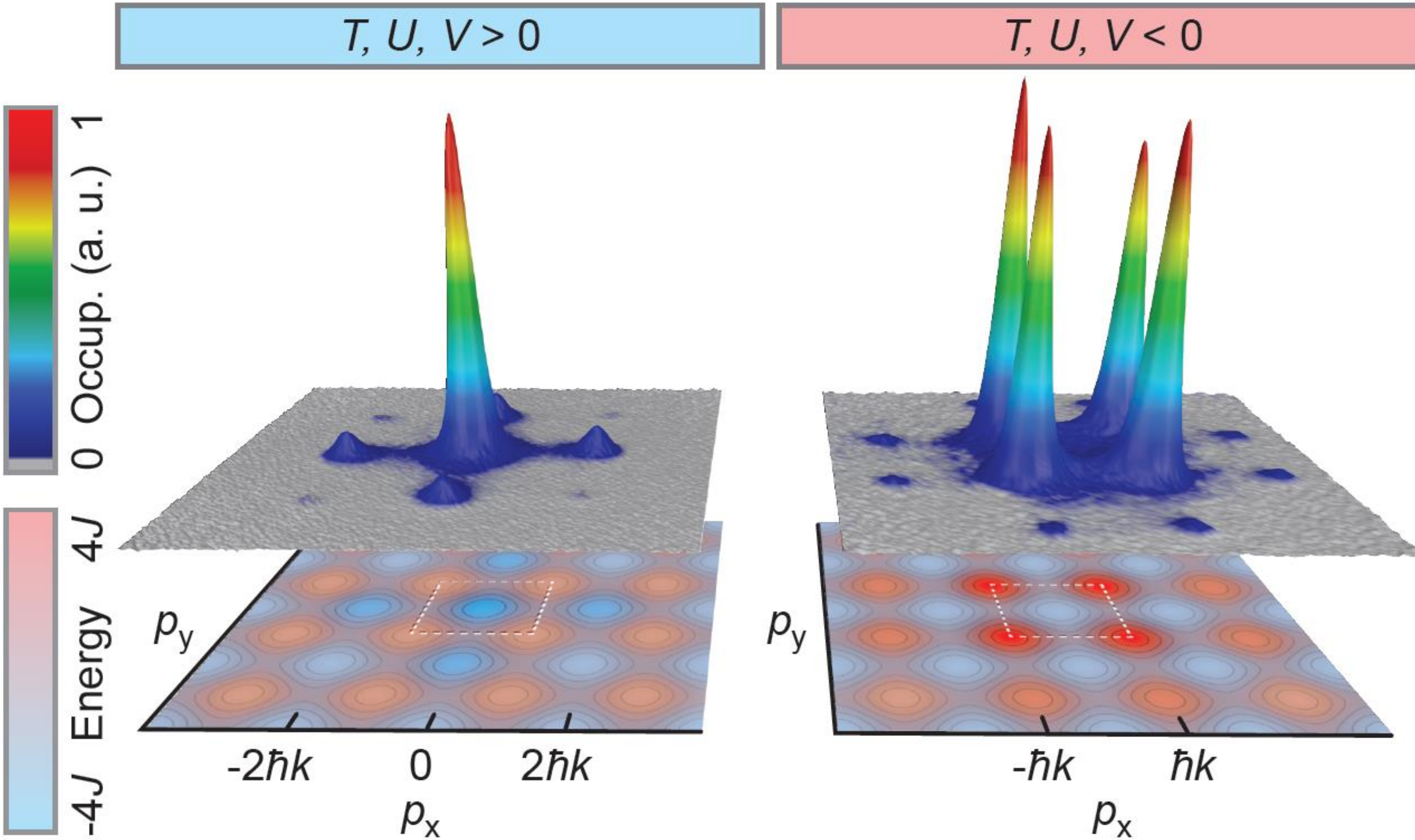
Proposal by:
Allard P. Mosk PRL **95**, 040403 (2005)
A. Rapp, S. Mandt and A. Rosch PRL **105**, 220405 (2010)

Bose gases at T>0 and T<0



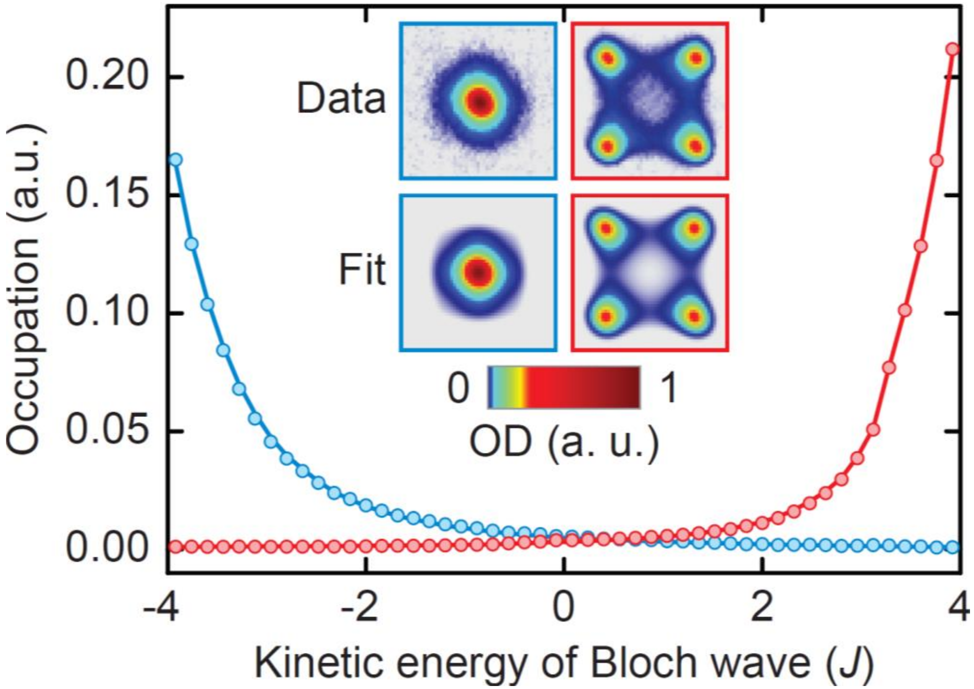
Science **339**, 52 (2013)

Bose gas at pos. and negative Temperature



Science **339**, 52 (2013)

Kinetic energy distribution



Blue: $T_{fit} = 2.7\,\frac{J}{k_B}$

Red: $T_{fit} = -2.2\,\frac{J}{k_B}$

Critical temperature in 2D:

- homogeneous, interacting

$$|T_{BKT}| \approx 1.8\,\frac{J}{k_B}$$

- trapped, non-interacting

$$|T_c| = 3.4(2)\,\frac{J}{k_B}$$

Kinetic energy distribution is well fitted by Bose-Einstein distribution.
→ System is (locally) in thermal state.

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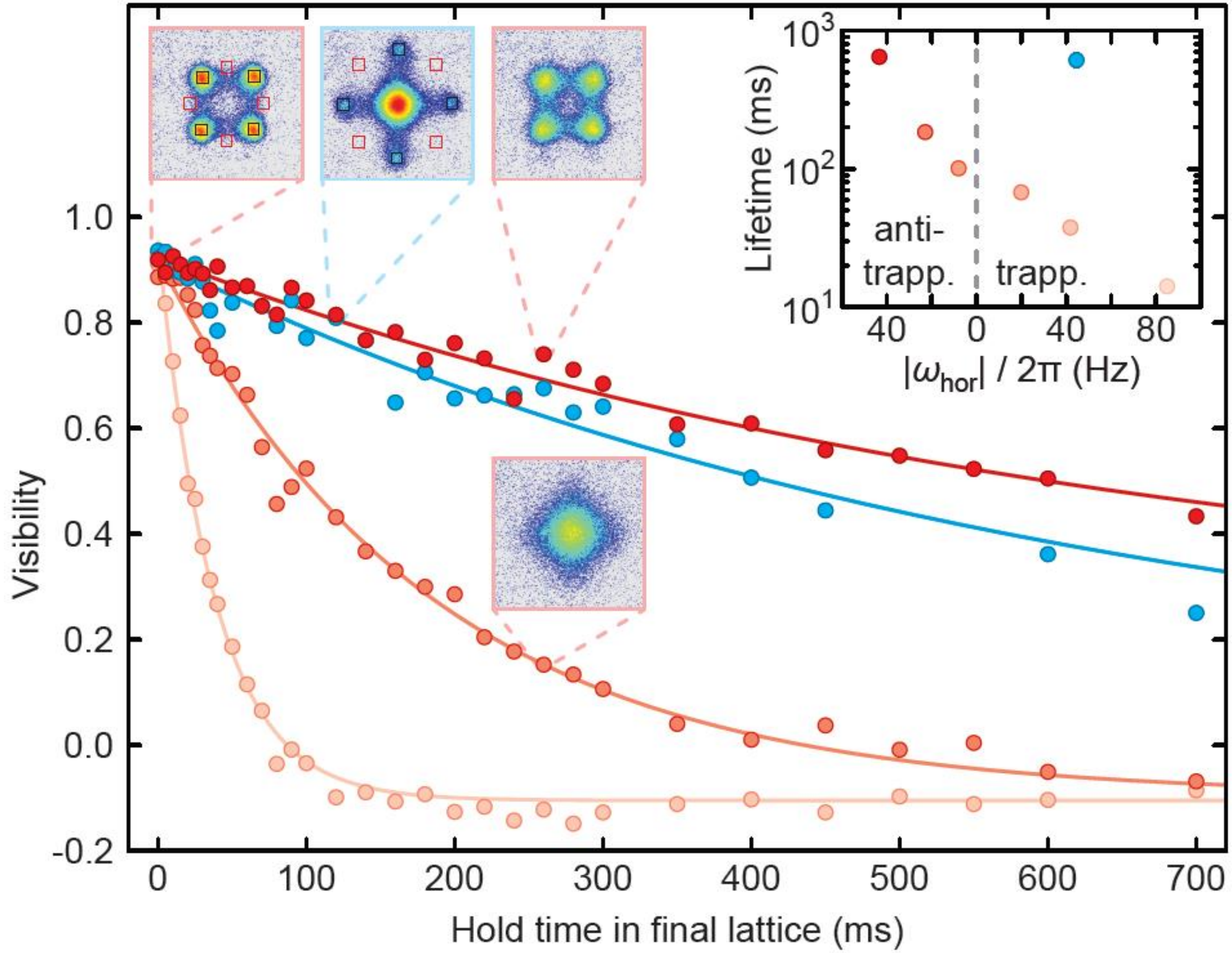
Lifetime of Negative Temperature state



Negative Temperature at U<0 as stable as positive T at U>0

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Stability?

$$\left.\frac{\partial S}{\partial V}\right|_E \geq 0$$

Stability criterium

Landau-Lifshitz, Vol5

- Container only gives upper limit for volume
- Thermal state = maximum entropy state (for given constraints)

$$dE = T\,dS - p\,dV \qquad dS = \frac{1}{T}\,dE + \frac{p}{T}\,dV \qquad \left.\frac{\partial S}{\partial V}\right|_E = \frac{p}{T}$$

$$\frac{p}{T} > 0$$

$T > 0 \;\Rightarrow\;$ positive pressure

$T < 0 \;\Rightarrow\;$ negative pressure

Negative Temperature stabilizes attractive Bose gas
➔No Bose-Nova

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Second law

- *Entropy*

  In an isolated system, entropy will only increase.

- *Clausius*

  Heat does not flow spontaneously from colder to hotter system.

- *Kelvin-Planck (original)*

  It is impossible to extract heat from a thermal reservoir and transform it completely into work. *Violated* ☹

- *Kelvin-Planck (modified)*

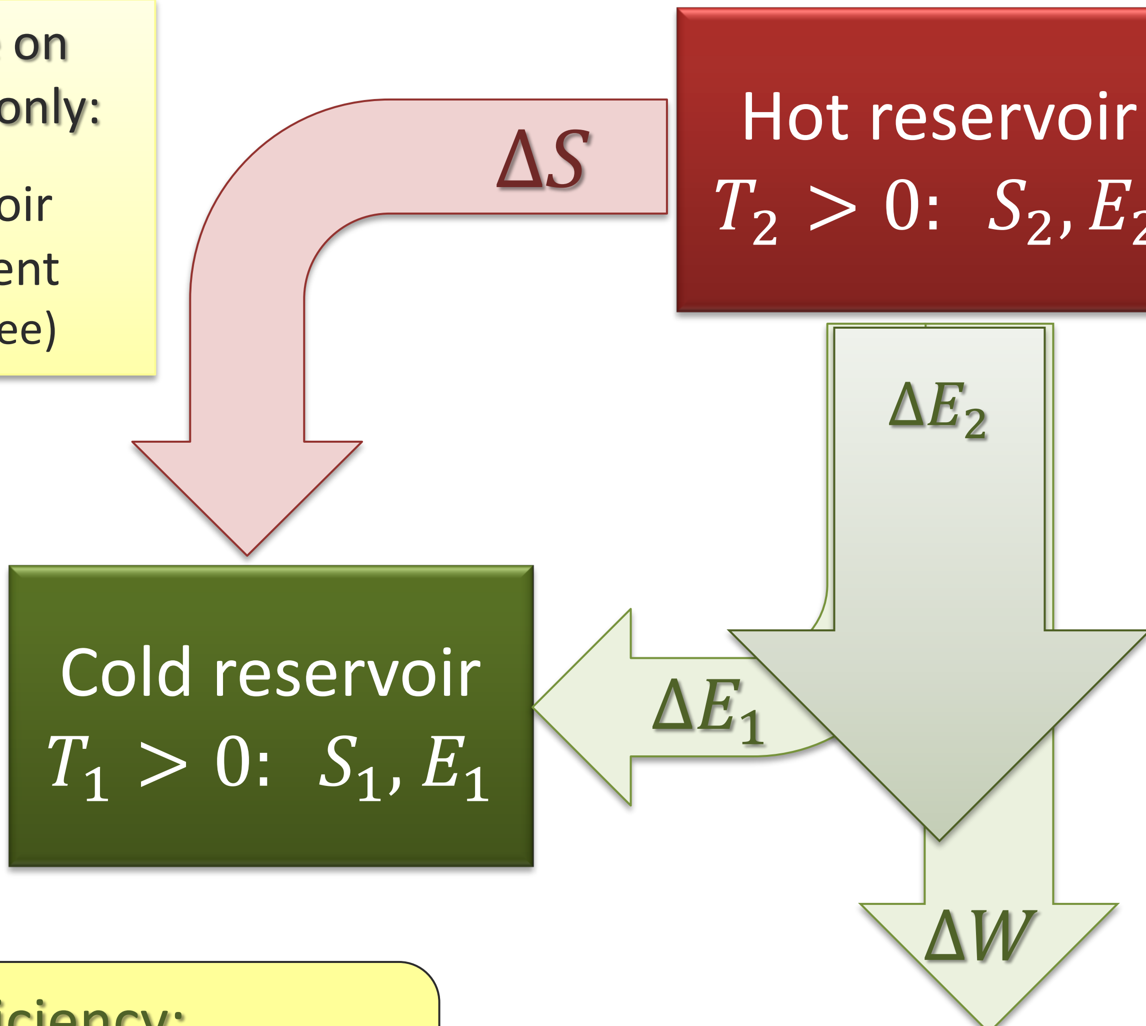
  It is impossible to extract heat from a positive temperature reservoir and transform it completely into work,
  and
  it is impossible to transfer work into heat that is completely inserted into a negative temperature reservoir. ***Ok*** ☺

Ramsey, Physical review (1956)

Carnot limit for positive temperatures

Concentrate on hot reservoir only:

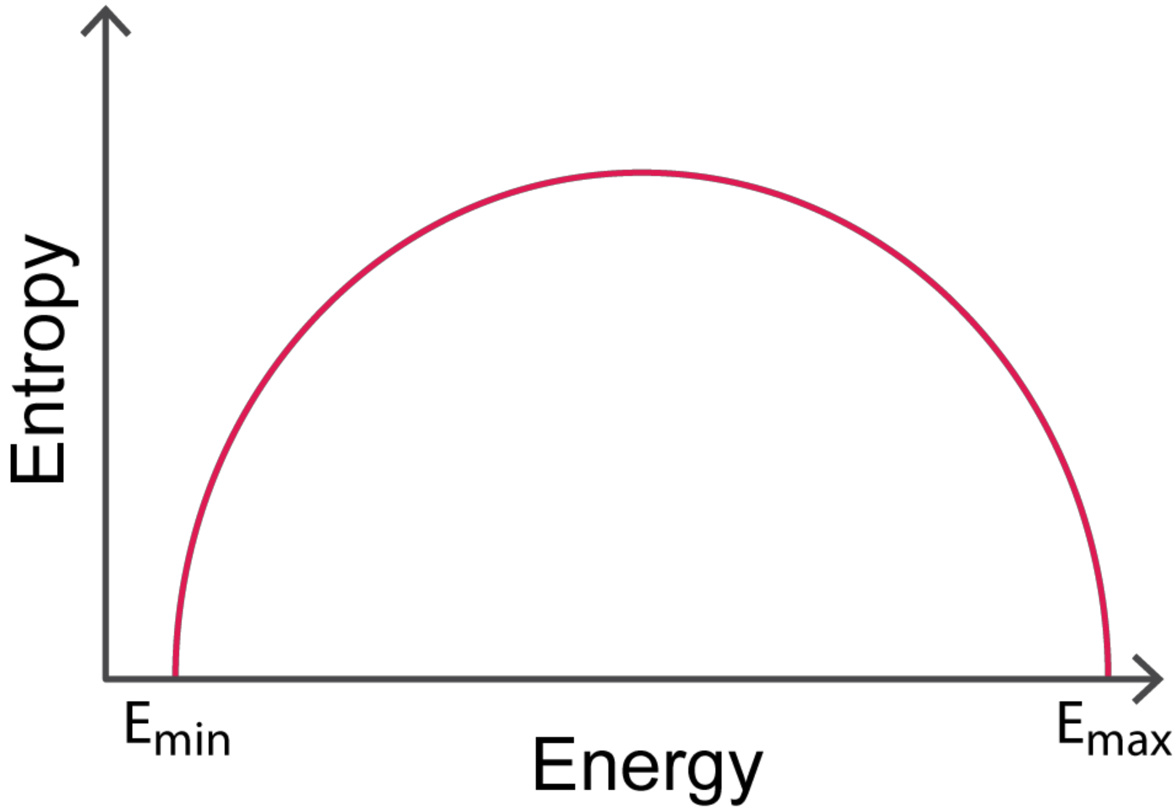
Cold reservoir = environment (comes for free)



Carnot Efficiency:

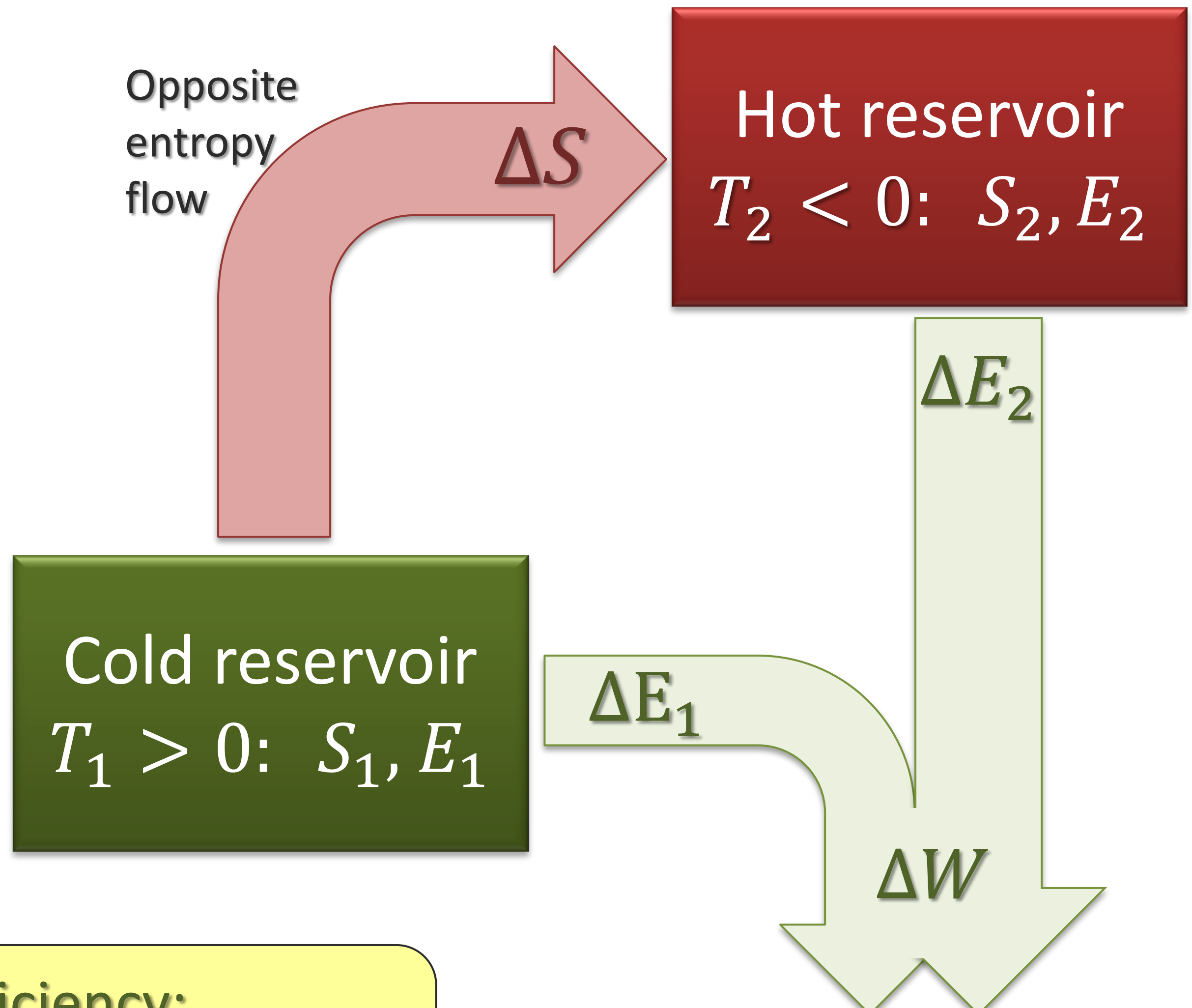
$$\Delta W = \Delta E_2 - \Delta E_1 \qquad \eta = \frac{\Delta W}{\Delta E_2} \leq 1$$

Energy-Entropy relation



$T \to 0^+$	$T \to \pm\infty$	$T \to 0^-$
$-\beta \to -\infty$	$-\beta \to 0$	$-\beta \to +\infty$

Carnot limit including negative Temperatures



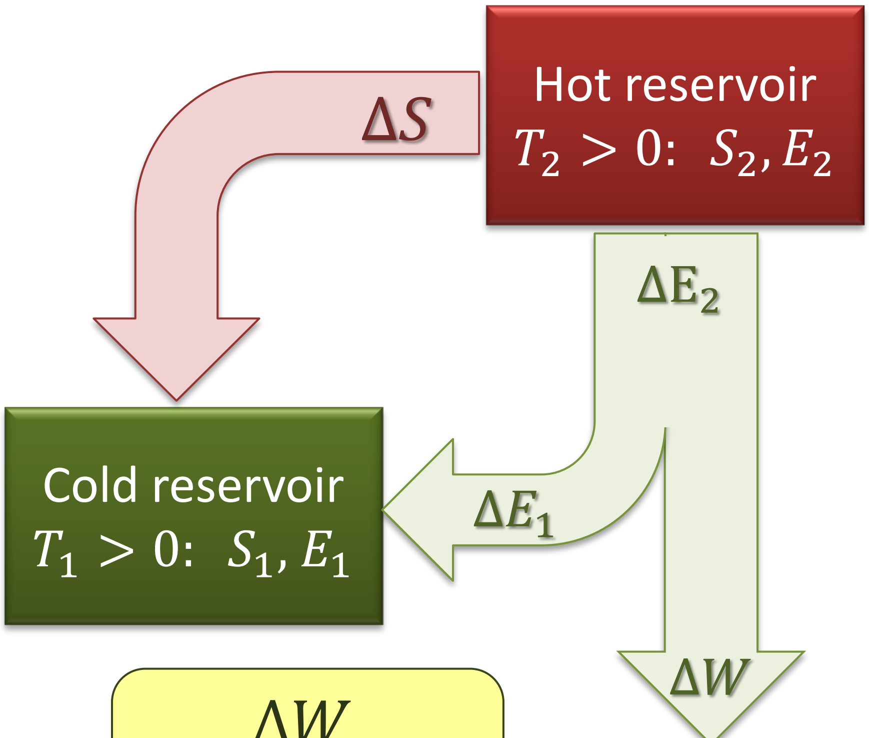
Carnot Efficiency:

$$\Delta W = \Delta E_2 + \Delta E_1 \qquad \eta = \frac{\Delta W}{\Delta E_2} > 1$$

Note: No quasi-static process can connect T<0 and T>0 ➔ Carnot-Engine is impossible

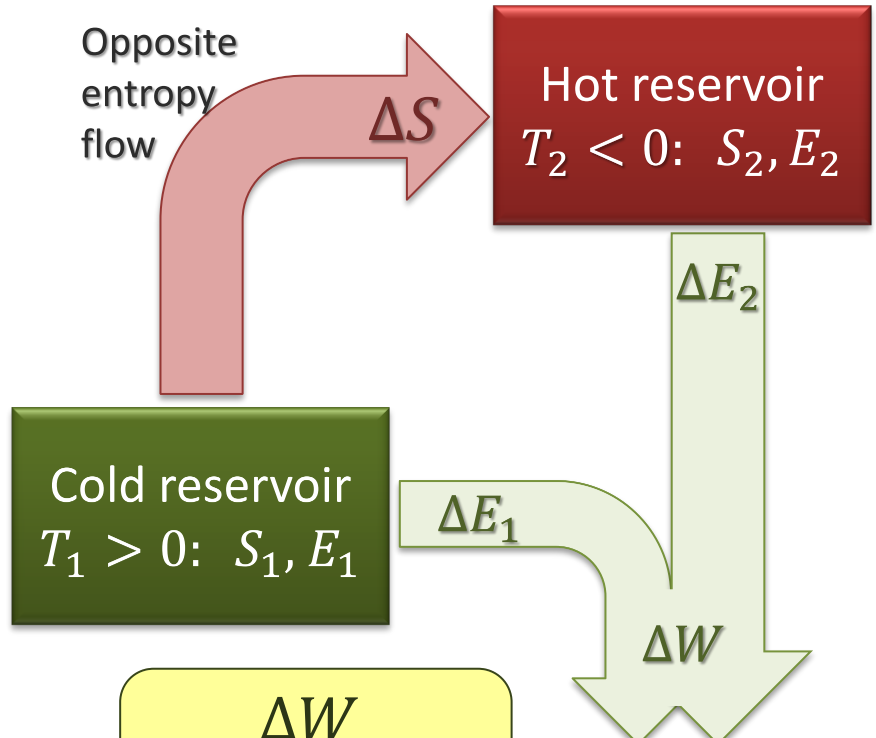
Carnot limits

$T_1 > 0, T_2 > 0$



$$\eta = \frac{\Delta W}{\Delta E_2} \leq 1$$

$T_1 > 0, T_2 < 0$



$$\eta = \frac{\Delta W}{\Delta E_2} > 1$$

- Energy and Entropy are globally conserved!
- No violation of thermodynamic laws → No solution to energy problem!

Applications

$$\hat{\rho} = e^{-\frac{\hat{H}}{k_B T}} = e^{-\frac{(-\hat{H})}{k_B(-T)}}$$

- *Bipartite lattices*: (e.g. simple cubic, hexagonal)

$$J \to -J \quad \hat{=} \quad k \to k + Q \quad (Q = (\pi,\pi,\pi)/d)$$

attractive **model at** ***T<0*** **=** ***repulsive*** **model at** ***T>0***

e.g. simulate attractive SU(3) model with ^{173}Yb

A. Rapp *PRA* **85**, 043612 (2012)

- *Non-Bipartite lattices*: (e.g. triangular, Kagome)

  → New many-body systems

  e.g. stabilize Bosons in flat band of Kagome lattice

  (Stamper-Kurn setup)

Summary

- Bose Gas at negative absolute Temperatures
  - realized for $U<0$, $V_0<0$

- Thermodynamically stable state with $T<0$, $p<0$

  → No collapse!

- Enables above unity Carnot efficiency

  $\eta = \frac{\Delta W}{\Delta E_2} > 1$ but **No** perpetuum mobile

  (E, S still conserved!)

- New many-body systems:
  - e.g. in Kagome lattice, attractive SU(3) in ^{173}Yb,

Ulrich Schneider LMU & MPQ Munich

Science **339**, 52 (2013)